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DESIGN AND ANALYSIS OF MULTI-SENSOR SEQUENTIAL DETECTION SYSTEM

University of Virginia

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A multiple sensor system is considered under binary hypothesis environments. All sensors are assumed independent, and the observed data is also independent. Received data is quantized and then sent to the fusion center to determine whether a target is present. The Sequential Probability Ratio Test is employed in the fusion center. The objective is to find an optimal system by minimizing the expected number of observations. Both two-level and four-level quantizer are used in the process of finding the optimal system. Numerical evaluations are made to find the quantizer which minimize the expected number of observations that are required to decide the presence of the target. System simulations are also performed to confirm the results.

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Abstract

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Table of Contrnts

List of Figures

List of Tables

CHAPTER 1

Introduction

1.1. Literature Review and Goals

The Sequential Probability Ratio Test (SPRT) was first developed by Wald. In [1], he presented the general theory of the sequential analysis and SPRT. Over the years, there are many books have been written about the sequential analysis. In particular, Dr. Siegmund has discussed this topic in a very precise and condensed form in his book [2].

In [3], sequential detection based on simple quantization has been analyzed. In this paper, the classical ruin problem in probability theory was applied to analyze the sequential dead-zone limiter detector and the sequential four-level detector. However, the author did not study the the effects of the quantization level of a quantizer on the sequential system, and his results were only applied to a single sensor. Chair, Hoballah and Varshney [4] applied the sequential detection theory to decentralized system where multisensors were used. A global decision was made based on the local decision of each sensor. Local decision rules were given by the likelihood ratio test (LRT), and the authors used the Neyman-Pearson approach to derive the optimal rules for each detector.

In our research, we also considered the problem of the distributed sequential detection system. However, instead of using LRT at each sensor as mentioned in Chair, Hoballah, and Varshney's paper, we simply quantize the observations into m-

level and sent them to the fusion center, the fusion center employs a sequential process that has the option to make a final decision on whether or not to continue the process by taking one more observation from each sensor. Our goals were to obtain the analytical expression on the expected number of samples that are required to make a decision in terms of the probabilities associated with the quantized observations, and to investigate the effects of different quantizers on the expected number. We are particularly interested in the systems which have two-level and four-level quantizers. The performances are compared through numerical evaluations and system simulations.

1.2. Overview of Chapters

Chapter 1 contains a literature review in which some of the important papers are discussed. It also describes the goals of the study, and the general overview of each chapter is given. The conditions and general assumptions are also mentioned.

A discussion on a two-sensor-system with m-level quantizer is presented in chapter 2. The sequential probability ratio test is stated for the system. The expression of the expected number of observations is also determined, and G function is defined in terms of expected number of observations.

In chapter 3, the emphasis is on the two-sensor system with two-level quantizer. By assuming that the distribution function of the observations has a double exponential character, the numerical evaluations are performed on the expected number of the system for various quantizers, and the optimal systems are found. the effects of the parameters of the double exponential on the system are also considered. The results

are plotted, and the evaluation program is in appendix A.

Work similar to that done in the previous chapter is done for the system with four-level quantizer in chapter 4. The analytical expression as well as the numerical evaluation of the expected number are performed for different four-level quantizers and parameters of the double exponential distribution function. The data is plotted in the end of the chapter, and the program of this study is in appendix B.

The results from chapter 3 and chapter 4 are presented in chapter 5. Each system's results are compared to the other results. The detailed discussion of each system is also given in this chapter.

In chapter 6, the optimal quantizers that are found in previous chapters are used to simulate the two-sensor system. The results are listed and plotted. In this chapter, the derivation of the random environment of double exponential is also discussed. The results are shown in the end of the chapter, and the simulation program is in appendix C.

In chapter 7, final comments and conclusions are made on the study.

1.3. Environment

The environment consists two hypothesis H_1 and H_0. H_1 shows that a target is present, and H_0 indicates that no target is present.

1.4. Assumption

We assume that the data observed at a sensor is independent and identical distributed. It is also assumed that the observations of one sensor are independent from

the others.

1.5. Advantages of the Sequential Test

The sequential test is considered optimal in the sense that it minimizes the expected sample size both under H_1 and H_0 among all tests have no larger error probability with independent and identical distributed observation [2]. Its average sample size is smaller than the fixed sample size required by the Neyman-Pearson test for the same performance.

CHAPTER 2

General Analysis of a Two-Sensor System

2.1. The Model and Configuration

2.1.1. Decision-Making at the Individual Sensor

There are two hypotheses H_1 and H_0. At each sensor, the hypotheses are presented by the probability density functions. The sensor quantizes the incoming data into m-levels. The m-level quantizer shown in Figure 2.1 is precalculated according to the optimal rule which will be derived in later chapters. As shown in Figure 2.2, each quantized level is associated with two conditional probabilities. If X_i falls between a_k and a_{k-1}, the probabilities associate with X_i are p_{K1} and p_{K0}. Figure 2.3 shows the quantized regions under hypotheses H_1 and H_0 of sensor two. The total number of bits that is transmitted to the fusion center depends on the number of the quantized level. For a four-level quantizer, it requires two bits of data.

2.1.2. Decision-Making at the Fusion Center

At the fusion center, Sequential Probability Ratio Test (SPRT) is employed. The decision is made by comparing the two predetermined thresholdes A and B where A is less than B. The fusion center will request more information from the sensors if it cannot decide whether the target is present.

The m-level Quantizer

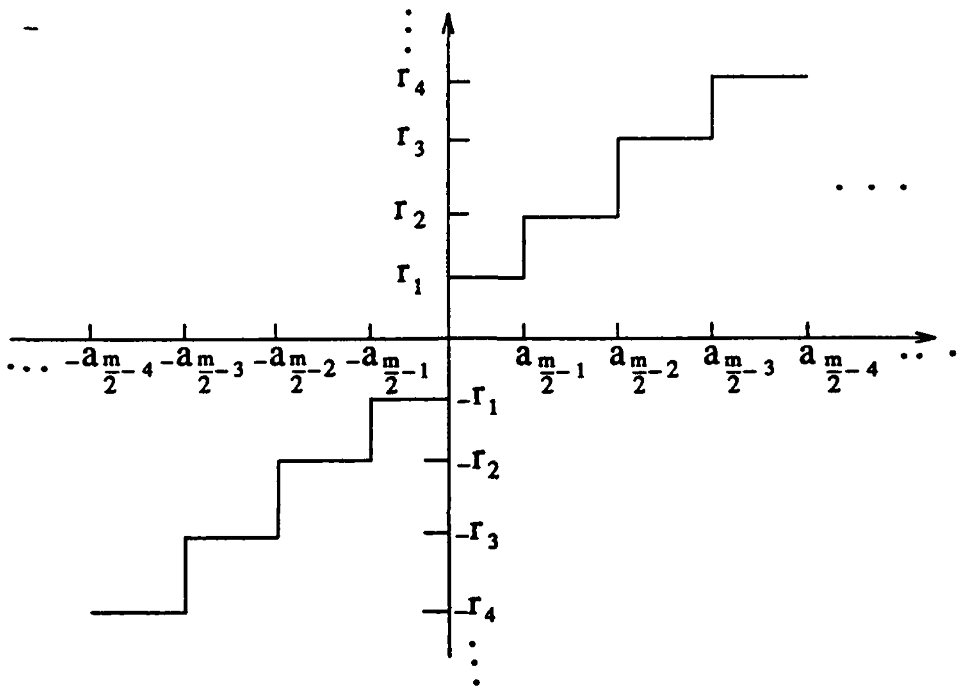

Figure 2.1 Model of m-level Quantizer

Quantized Regions for Sensor 1

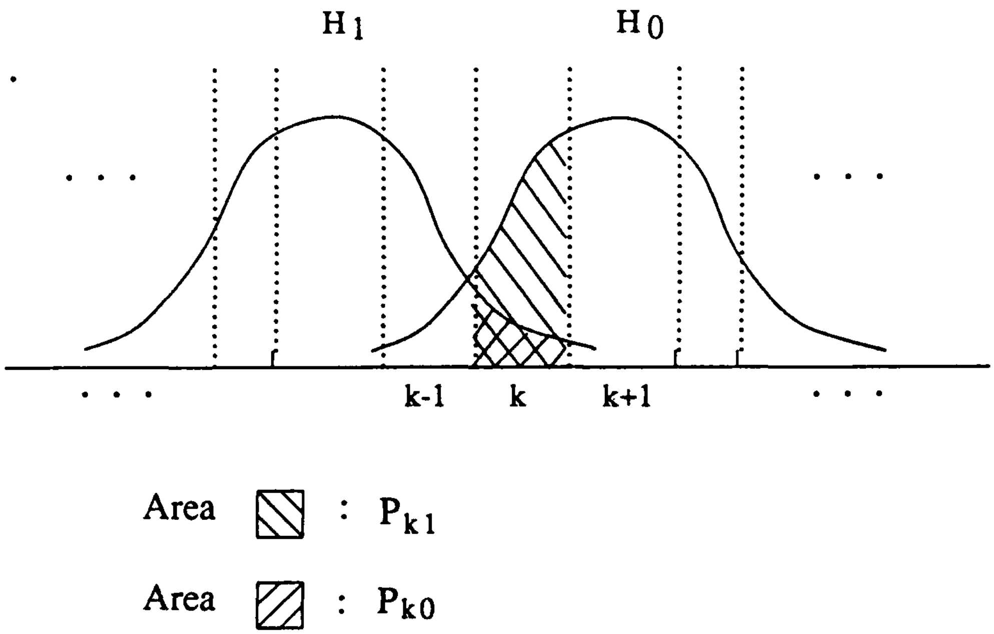

Figure 2.2 The m-level Quantized Probability Space of Sensor 1

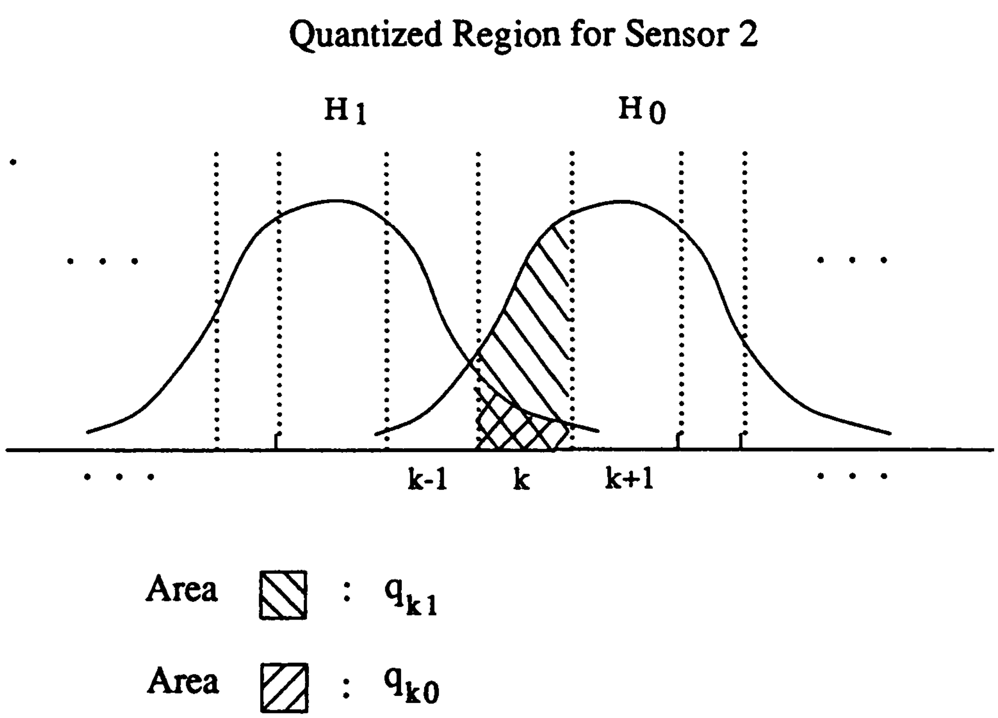

Figure 2.3 The m-level Quantized Probability Space of Sensor 2

2.1.3. The Overall Process of the Model

As mentioned in chapter 1, sensors are independent each other. The incoming signals are first quantized into m-levels, and then send to the fusion center. Depending on whether the likelihood ratio(LR) at the fusion center is greater than B or less than A, the final decision is made. A target is either detected if LR is greater than B, or the target is not present if LR is less than A. If the LR is laid in between the thresholds A and B, the center requests each sensor to send one more quantized observation as shown in Figure 2.4.

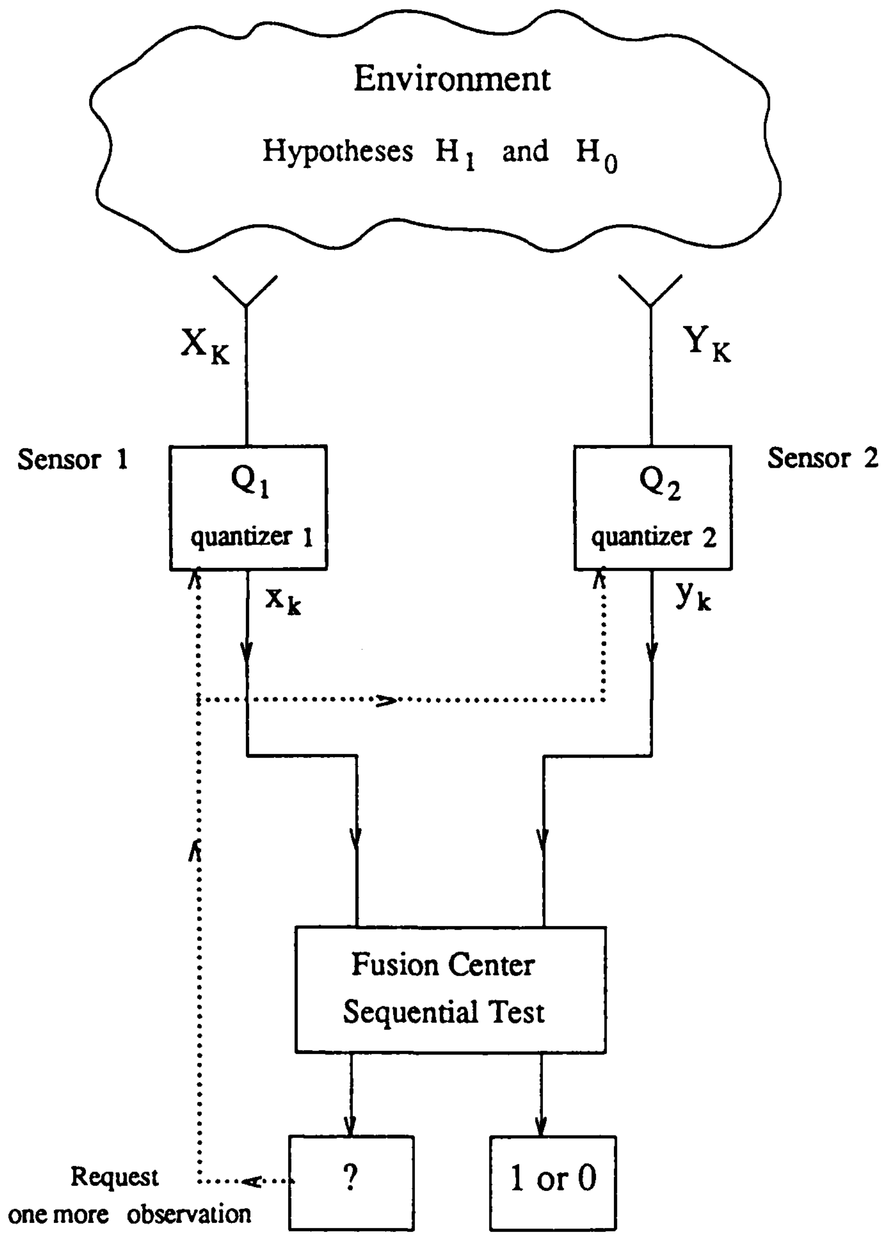

Figure 2.4 Model of the Sequential Detection System

2.2. Definition of the Sequential Probability Ratio Test

2.2.1. General Definition of SPRT

If $x_1,x_2,...$ is a sequence of random variables with join density function fn, and there are two hypotheses H_1 and H_0 such that:

$$H_0: \quad f_n = f_{0n}(x_1,x_2,\ldots,x_n) \tag{2.2.1}$$

$$H_1: \quad f_n = f_{1n}(x_1,x_2,\ldots,x_n) \tag{2.2.2}$$

l_n is defined as likelihood ratio:

$$\begin{aligned} l_n &= l_n(x_1,x_2,\ldots,x_n) \\ &= \frac{f_{1n}(x_1,x_2,\ldots,x_n)}{f_{0n}(x_1,x_2,\ldots,x_n)} \end{aligned} \tag{2.2.3}$$

Let the thresholds A and B are chosen such that $0 < A < B < \infty$, and start observing data $x_1,x_2,\ldots,$ sequentially until the random time N which is the first n such that either $l_n \geq B$, or $l_n \leq A$. If $l_n \geq B$, then H_1 is selected, and if $l_n \leq A$, then H_0 is selected. This can be expressed mathematically as following:

$$N = \begin{cases} \text{first } n \geq 1 & \text{such that } l_n \notin (A,B) \\ \infty & \text{if } l_n \in (A,B) \text{ for all } n \geq 1 \end{cases} \tag{2.2.4}$$

Stop sampling at the time N and if $N < \infty$

$$\text{Reject } H_1 \quad \text{if } l_n \geq B$$

$$\text{Accept } H_0 \quad \text{if } l_n \leq A$$

2.2.2. Calculation of the Thresholds

Thresholds A and B were calculated in terms of the error probabilities of first and second kind (α and β, respectively) in [1].

For given α and β where

$$\alpha = P_0\{l_N \geq B\} \tag{2.2.5}$$

$$\beta = P_1\{l_N \leq A\} \tag{2.2.6}$$

A and B can be expressed as following:

$$A = \frac{1-\beta}{\alpha} \tag{2.2.7}$$

$$B = \frac{\beta}{1-\alpha} \tag{2.2.8}$$

2.3. SPRT for Two Distributed Sensors

2.3.1. Definition of the LR

Extending the definition of the LR in (2.2.3) to two sensors, the LR of this system can be written as:

$$l_n = \frac{f_{1n}(x_1, x_2, \ldots, x_n;\, y_1, y_2, \ldots, y_n)}{f_{0n}(x_1, x_2, \ldots, x_n;\, y_1, y_2, \ldots, y_n)} \tag{2.3.1}$$

where $f_{1n}(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_n)$ and $f_{0n}(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_n)$ are join density function of observed random sequence $x_1, x_2, \ldots, x_n$ from sensor 1 and $y_1, y_2, \ldots, y_n$ from sensor 2 under hypotheses H_1 and H_0 respectively (see Figure 2.3).

Applyng the assumption that the observations of one sensor are independent from the other, the above equation (2.3.1) can be re-written as:

$$l_n = \frac{f_{1n}(x_1, x_2, \ldots, x_n) f_{1n}(y_1, y_2, \ldots, y_n)}{f_{0n}(x_1, x_2, \ldots, x_n) f_{0n}(y_1, y_2, \ldots, yn)} \tag{2.3.2}$$

Furthermore, if the individual observation of each sensor is independently distributed, (2.3.2) becomes:

$$l_n = \prod_{k=1}^{n} \frac{f_{1n}(x_k) f_{1n}(y_k)}{f_{0n}(x_k) f_{0n}(y_k)} \tag{2.3.3}$$

Taking log on both sides to arrive at the log likelihood ratio:

$$\log l_n = \sum_{k=1}^{n} \log \frac{f_{1n}(x_k) f_{1n}(y_k)}{f_{0n}(x_k) f_{0n}(y_k)} \tag{2.3.4}$$

2.3.2. SPRT

Using the LR that is defined in the previous section, the SPRT can be stated as follows:

Sampling the random sequence $x_1, x_2, \ldots$ and $y_1, y_2, \ldots$ sequentially until the random time N, such that $\log l_N$ is greater or equal to b, or less than and equal to a. The hypothesis H_1 is accepted if $\log l_N \geq b$ and the hypothesis H_0 is accepted if $\log l_N \leq a$, where

$$a = \log A'$$

$$b = \log B'$$

A' and B' will be derived in the next section.

2.3.3. Calculation of the Thresholds A' and B'

As defined in equation (2.2.5), α is the error probability of first kind, and

$$\begin{aligned} \alpha &= \text{Prob}\{\text{ decide } H_1 \mid H_0\} \\ &= \sum_{n=1}^{\infty} \text{Prob}\{\text{ stop at n and decide } H_1 \mid H_0\} \\ &= \sum_{n=1}^{\infty} P_0(N = n,\ l_n \geq B') \end{aligned} \tag{2.3.6}$$

Let B_n be the subset of n-dimensional space where $A' < l_k(x_1, \ldots, x_k; y_1, \ldots, y_k) < B'$ for every $k < n$, and

$l_n(x_1,\ldots,x_n;y_1,\ldots,y_n) \geq B'$. Hence, the set $\{N=n,\ l_n > B'\} = \{(x_1,\ldots,x_n;y_1,\ldots,y_n) \in B_n\}$.

Let A_n be the subset of n-dimensional space where $A' < l_n(x_1,\ldots,x_k;y_1,\ldots,y_k) < B'$ for every $k<n$ and $l_n(x_1,\ldots,x_n;y_1,\ldots,y_n) \leq A'$. Hence the set $\{N=n,\ l_n \leq A'\} = \{(x_1,\ldots,x_n;y_1,\ldots,y_n) \in A_n\}$.

Now the definition in equation (2.3.6)

$$\alpha = \sum_{n=1}^{\infty} \int_{x_1,\ldots,x_n,y_1,\ldots,y_n \in B_n} f_{0n}(x_1,\ldots,x_n,y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n$$

$$= \sum_{n=1}^{\infty} \int_{B_n} f_{0n}(x_1,\ldots,x_n)f_{0n}(y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n. \tag{2.3.7}$$

Using the condition that $l_n \geq B'$,

$$\alpha \leq \frac{1}{B'}\sum_{n=1}^{\infty}\int_{B_n} f_{1n}(x_1,\ldots,x_n)f_{1n}(y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n$$

$$= \frac{1}{B'}\sum_{n=1}^{\infty}\int_{B_n} f_{1n}(x_1,\ldots,x_n,y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n$$

$$= \frac{1}{B'}\sum_{n=1}^{\infty} P_1\{N=n,\ l_n \geq B')$$

$$= \frac{1}{B'}\text{Prob}\{\text{ decide } H_1 \mid H_1\}$$

$$= \frac{1}{B'}(1-\text{Prob}\{\text{ decide } H_0 \mid H_1\})$$

$$= \frac{1-\beta}{B'}. \tag{2.3.8}$$

By the same method,

$$\beta = \text{Prob}\{\text{ decide } H_0 \mid H_1\}$$

$$= \sum_{n=1}^{\infty} \text{Prob}\{\text{ stop at n and decide } H_0 \mid H_1\}$$

$$= \sum_{n=1}^{\infty} P_1(N = n.\ l_n \le A')$$

$$= \sum_{n=1}^{\infty} \int_{x_1,\ldots,x_n,y_1,\ldots,y_n \in A_n} f_{1n}(x_1,\ldots,x_n,y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n$$

$$= \sum_{n=1}^{\infty} \int_{A_n} f_{1n}(x_1,\ldots,x_n) f_{1n}(y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n.$$

$$\le A' \sum_{n=1}^{\infty} \int_{A_n} f_{0n}(x_1,\ldots,x_n) f_{0n}(y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n$$

$$= A' \sum_{n=1}^{\infty} \int_{A_n} f_{0n}(x_1,\ldots,x_n,y_1,\ldots,y_n)dx_1\cdots dx_n dy_1\cdots dy_n$$

$$= A' \sum_{n=1}^{\infty} P_0\{ N = n,\ l_n \le A')$$

$$= A'\text{Prob}\{\text{ decide } H_0 \mid H_0\}$$

$$= A'(1 - \text{Prob}\{\text{ decide } H_1 \mid H_0\})$$

$$= (1-\alpha)A'. \tag{2.3.9}$$

The α and β can be approximated. Since the equations (2.3.8) and (2.3.9) have inequalities only because l_n does not have to reach the boundaries A' and B' exactly when it first leave (A',B'), we can ignore this discrepency and treat (2.3.8) and (2.3.9) as equalities. Then

$$\alpha \approx \frac{1-\beta}{B'} \tag{2.3.10}$$

$$\beta \approx A'(1-\alpha) \tag{2.3.11}$$

2.3.4. Derive the Expected Value of N ($E_i\{N\}$)

To derive $E_i\{N\}$, it is necessary to look at the equation (2.3.4). By taking the expected value on both sides of the equation, it becomes:

$$E_i \log l_n = E_i\left\{\sum_{k=1}^{n} \log \frac{f_{1n}(x_k) f_{1n}(y_k)}{f_{0n}(x_k) f_{0n}(y_k)}\right\} \tag{2.4.1}$$

where $E_i\{\ \}$ is the conditional expectation under hypothses H_i, and $i = 0, 1$.

Wald's identity [1] said that if $V_1, V_2, \ldots$ is i.i.d with mean value $\mu = EV_1$, Let M be any integer valued random variable such that $(M = n)$ is an event determined by conditions on $V_1, V_2, \ldots, V_n$ for all $n = 1,2,\ldots$, and assume that $EM < \infty$, then $E(\sum_{k=1}^{M} V_k) = \mu EM$.

From (2.4.1), we can define V_k as:

$$V_k = \log \frac{f_{1n}(x_k) f_{1n}(y_k)}{f_{0n}(x_k) f_{0n}(y_k)} \tag{2.4.2}$$

Since $f_{1n}(x_k), f_{1n}(y_k), f_{0n}(x_k)$, and $f_{0n}(y_k)$ are all i.i.d, $\log \frac{f_{1n}(x_k) f_{1n}(y_k)}{f_{0n}(x_k) f_{0n}(y_k)}$ is also i.i.d. Using the definition of Wald's identity on the right side of the equation (2.4.1), it gives

$$E_i \log l_n = \mu_i E_i\{N\} = E_i V_1 E_i\{N\} \tag{2.4.3}$$

On the other hand, the left side of the equation (2.4.1) can also be approximated as a two-valued random variable taking on the values a and b by the same reasoning used to derive (2.3.10) and (2.3.11).

$$E_i \log l_n = aP_i(l_n \leq A') + bP_i \tag{2.4.4}$$

After applying equations (2.3.10) and (2.3.11), and doing some simplifications, the equation (2.4.4) can be re-written as:

$$E_0 \log l_n = a(1-\alpha) + b\frac{1-\beta}{B'}$$

$$= a\left(1 - \frac{1-A'}{B'-A'}\right) + b\frac{1-A'\dfrac{B'-1}{B'-A'}}{B'}$$

$$= \frac{a(B'-1)+b(1-A')}{B'-A'} \tag{2.4.5}$$

and

$$E_1 \text{Log}\, l_n = aA'(1-\alpha) + b(1-\beta)$$

$$= \frac{aA'(B'-1)+bB'(1-A')}{B'-A'} \tag{2.4.6}$$

By substitution, equation (2.4.3) can be written as:

$$E_1 N = E_1^{-1}\{V_1\}\frac{aA'(B'-1)+bB'(1-A')}{B'-A'} \tag{2.4.7}$$

$$E_0 N = E_0^{-1}\{V_1\}\frac{a(B'-1)+b(1-A')}{(B'-A')} \tag{2.4.8}$$

2.4. Evaluate $E_i\ \{N\}$ Under Quantization

As mentioned in section 2.1, the system described here utilizes a quantizer at each sensor to quantize the received signals. Assuming a m-level quantizer is used, thus the space is divided into m intervals. At the jth interval of sensor 1, there are two conditional probabilities associated with it, namely $P(j \mid H_0)$ and $P(j \mid H_1)$. The general conditional probability of both sensors are defined below:

For sensor 1: $P(x_1 = j \mid H_1) = p_{j1}$ (2.5.1)

$$P(x_1 = j \mid H_0) = p_{j0} \tag{2.5.2}$$

For sensor 2: $P(y_1 = l \mid H_1) = q_{l1}$ (2.5.3)

$$P(y_1 = l \mid H_0) = q_{l0} \tag{2.5.4}$$

where l, j = 1,2,...,m. Using these definitions in (2.5.1), (2.5.2), (2.5.3), and (2.5.4), the equation (2.4.2) for V_1 can be re-written as follows:

$$V_1 = \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}} \tag{2.5.5}$$

Thus $E_i\{V_1\}$ becomes

$$\begin{aligned} E_i\{V_1\} &= E_i\{\log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}}\} \\ &= \sum_j \sum_l P(x_1 = j, y_1 = l \mid H_i) \log \frac{p_{j.} q_{l1}}{p_{j0} q_{l0}} \\ &= \sum_j \sum_l P(x_1 = j \mid hi) P(y_1 = l \mid H_i) \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}} \\ &= \sum_j \sum_l p_{ji} q_{ji} i \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}} \end{aligned} \tag{2.5.6}$$

where i = 1,0; $j, l \in (1,\ldots,m)$ by invoking appropriate independence assumptions.

If the above equation is substituted into (2.4.7) and (2.4.8), the conditional expectation of N can be expressed as:

$$E_1 N = \frac{aA'(B'-1) + bB'(1-A')}{B'-A'} \left(\sum_j \sum_l p_{j1} q_{l1} \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}}\right) \tag{2.5.7}$$

$$E_0 N = \frac{a(B'-1) + b(1-A')}{(B'-A')} \left(\sum_j \sum_l p_{j0} q_{l0} \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}}\right) \tag{2.5.8}$$

2.4.1. Definition of the G Function

In the previous section, E_1N and E_0N have been evaluated. In order to find optimal solution for the entire system, we first defined a function G,

$$G = \sum_{i=0,1} E_i N.$$

Now we express the G function in terms of E_1N and E_0N:

$$G = \frac{aA'(B'-1)+bB'(1-A')}{B'-A'}\left(\sum_j \sum_l p_{j1} q_{l1} \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}}\right)$$

$$G \quad + \frac{a(B'-1)+b(1-A')}{(B'-A')}\left(\sum_j \sum_l p_{j0} q_{l0} \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}}\right) \tag{2.5.9}$$

If α and β are given, then A' and B' are known from equations (2.3.8) and (2.3.9). The G function only depend on the probability of quantization levels. Thus, to have an optimal system, one must to find the optimal quantizers for both sensors that will minimize the equation G.

In next chapter, we will study the effect of the two-level quantizer on the system.

CHAPTER 3

Analysis of a Two-Sensor System with Two-level Quantizer

3.1. The Model and Configuration

The system that is analyzed here is closely related to the one in chapter two. However, in chapter two, the signals are quantized into m-level at each sensor. The main concern in this chapter is to study the two-level quantizer system both theoretically and numerically.

3.1.1. The Model of Sensor

Since sensor 1 and sensor 2 are identically process the incoming data, it is reasonable to choose an arbitrary sensor as an example. Taking a look at sensor 1, shown in the figure 3.1, a two-level quantizer with thresholds T_1 divides both probability functions $f_{11}(\,)$ and $f_{01}(\,)$ into two regions. The received signal X_k is quantized to 1 if it falls below the threshold T_1, and it is quantized to 2 otherwise. For the kth signal, the conditional probabilities associate with each quantized level are listed below:

For sensor 1:

$$p_{21}^k = P(X_k \text{ is above the threshold } T_1 \mid H_1) = P(x_k = 2 \mid H_1) \tag{3.1.1}$$

$$p_{11}^k = P(X_k \text{ is below the threshold } T_1 \mid H_1) = P(x_k = 1 \mid H_1) \tag{3.1.2}$$

$$p_{20}^k = P(X_k \text{ is above the threshold } T_1 \mid H_0) = P(x_k = 2 \mid H_0) \tag{3.1.3}$$

$$p_{10}^k = P(X_k \text{ is below the threshold } T_1 \mid H_0) = P(x_k = 1 \mid H_0) \tag{3.1.4}$$

Two-level Quantizer System

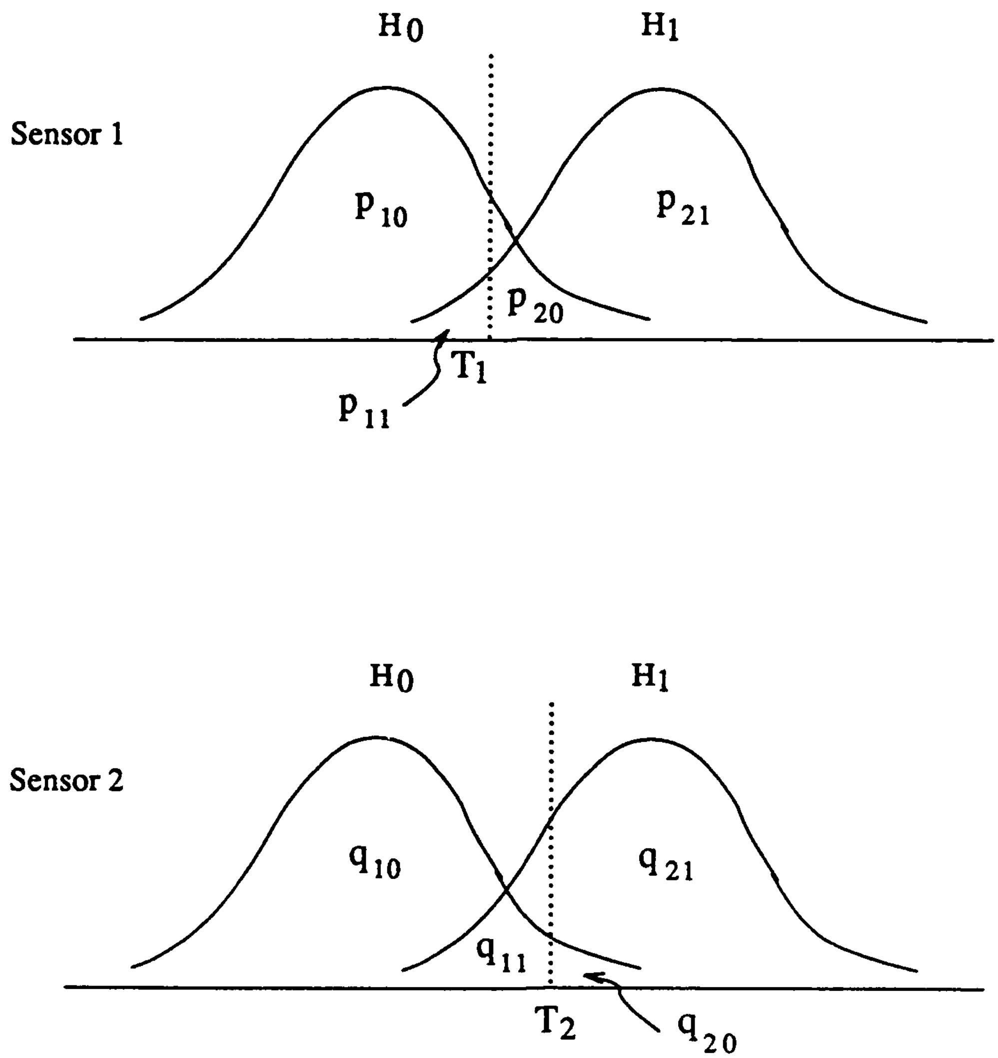

Figure 3.1 Quantized Probability Space of Two-level Quantizer System

For sensor 2:

$$q_{21}^{k} = P(\, Y_k \text{ is above the threshold } T_2 \mid H_1) = P(y_k = 2 \mid H_1) \tag{3.1.5}$$

$$q_{11}^{k} = P(\, Y_k \text{ is below the threshold } T_2 \mid H_1) = P(y_k = 1 \mid H_1) \tag{3.1.6}$$

$$q_{20}^{k} = P(\, Y_k \text{ is above the threshold } T_2 \mid H_0) = P(y_k = 2 \mid H_0) \tag{3.1.7}$$

$$q_{10}^{k} = P(\, Y_k \text{ is below the threshold } T_2 \mid H_0) = P(y_k = 1 \mid H_0) \tag{3.1.8}$$

3.2. Evaluate the Expected Value of N ($E_i\{N\}$)

The general expression of $E_i\{N\}$ has been evaluated in chapter 2.

To derive the equation of $E_i\{N\}$ for the system with two-level quantizer, one must find $E_i\{V_1\}$ first. Using the probabilities defined in the last section, the equation (2.5.6) can be expanded as:

$$\begin{aligned} E_i\{V_1\} &= \sum_j \sum_l p_{ji} qli \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}} \\ &= p_{1i} q_{1i} \log \frac{p_{11} q_{11}}{p_{10} q_{10}} + p_{1i} q_{2i} \log \frac{p_{11} q_{21}}{p_{10} q_{20}} \\ &\quad + p_{2i} q_{1i} \log \frac{p_{21} q_{11}}{p_{20} q_{20}} + p_{2i} q_{2i} \log \frac{p_{21} q_{21}}{p_{20} q_{20}} \end{aligned} \tag{3.2.1}$$

where

$p_{1i} = p_{1i}^{1};$

$p_{2i} = p_{2i}^{1};$

$q_{1i} = q_{1i}^{1};$

$q_{2i} = q_{2i}^{1};$

and $i = 0, 1$. After some simplifications,

$$E_1\{V_1\} = p_{21} \log \frac{p_{20} p_{11}}{p_{21} p_{10}} + q_{21} \log \frac{q_{20} q_{11}}{q_{21} q_{10}} + \log \frac{p_{10} q_{10}}{p_{11} q_{11}} \tag{3.2.2}$$

$$E_0\{V_1\} = p_{20} \log \frac{p_{20} p_{11}}{p_{21} p_{10}} + q_{20} \log \frac{q_{20} q_{11}}{q_{21} q_{10}} + \log \frac{p_{10} q_{10}}{p_{11} q_{11}} \tag{3.2.3}$$

Then the $E_i\{N\}$ can be expressed as follows:

$$E_1\{N\} = \frac{aA'(B'-1)+b(B'-A')}{B'-A'}\left[p_{21}\log\frac{p_{20}p_{11}}{p_{21}p_{10}} + q_{21}\log\frac{q_{20}q_{11}}{q_{21}q_{10}}\right.$$

$$\left. + \log\frac{p_{10}q_{10}}{p_{11}q_{11}}\right]^{-1} \tag{3.2.4}$$

$$E_0\{N\} = \frac{a(B'-1)+b(1-A')}{B'-A'}\left[p_{20}\log\frac{p_{20}p_{11}}{p_{21}p_{10}} + q_{20}\log\frac{q_{20}q_{11}}{q_{21}q_{10}}\right.$$

$$\left. + \log\frac{p_{10}q_{10}}{p_{11}q_{11}}\right]^{-1} \tag{3.2.5}$$

3.2.1. An Alternative to Derive $E_i\{N\}$

In the case of two-level quantizer, if the signal is quantized into -1 or 1 depending on whether it falls below or above the threshold T_1, then the signal is an antipodal signal. Let's define

$$S_n = x_1 + x_2 + \cdots + x_n \quad \text{for sensor 1} \tag{3.2.6}$$

$$\overline{S}_n = y_1 + y_2 + \cdots + y_n \quad \text{for sensor 2} \tag{3.2.7}$$

Notice here that $\{S_n\}$ is a case of random walk. If assuming a_i is the number of 1's among n signals, then $n - a_i$ is the number of -1's,

$$S_n = a_1 - (n - a_1) \tag{3.2.8}$$

$$\overline{S}_n = a_2 - (n - a_2) \tag{3.2.9}$$

Now expressing a_i and $n - a_i$ in terms of n, S_n, and $\overline{S}_n$:

$$a_1 = \frac{n + S_n}{2} \tag{3.2.10}$$

$$n - a_1 = \frac{n - S_n}{2} \tag{3.2.11}$$

$$a_2 = \frac{n + \bar{S}_n}{2} \tag{3.2.12}$$

$$n - a_2 = \frac{n - \bar{S}_n}{2} \tag{3.2.13}$$

In the section 3.1, it was mentioned that an incoming signal is quantized into 1 and 2. However, no matter what value we assign to the quantization level, the probability that associates with it is the same. Thus

$$P\{x_1 = 1 \mid H_0\} = p_{20} \tag{3.2.14}$$

$$P\{x_1 = -1 \mid H_0\} = p_{10} \tag{3.2.15}$$

$$P\{x_1 = 1 \mid H_1\} = p_{21} \tag{3.2.16}$$

$$P\{x_1 = -1 \mid H_1\} = p_{11} \tag{3.2.17}$$

$$P\{y_1 = 1 \mid H_0\} = q_{20} \tag{3.2.18}$$

$$P\{y_1 = -1 \mid H_0\} = q_{10} \tag{3.2.19}$$

$$P\{y_1 = 1 \mid H_1\} = q_{21} \tag{3.2.20}$$

$$P\{y_1 = -1 \mid H_1\} = q_{11}. \tag{3.2.21}$$

The LR becomes

$$\begin{aligned} l_n &= \frac{f_1(x_1,x_2,\ldots,x_n) f_1(y_1,y_2,\ldots,y_n)}{f_0(x_1,x_2,\ldots,x_n) f_0(y_1,y_2,\ldots,y_n)} \\ &= \frac{[P\{x_1 = 1 \mid H_1\}]^{a_1} [P\{x_1 = -1 \mid H_1\}]^{n-a_1} [P\{y_1 = 1 \mid H_1\}]^{a_2} [P\{y_1 = -1 \mid H_1\}]^{n-a_2}}{[P\{x_1 = 1 \mid H_0\}]^{a_1} [P\{x_1 = -1 \mid H_0\}]^{n-a_1} [P\{y_1 = 1 \mid H_0\}]^{a_2} [P\{y_1 = -1 \mid H_0\}]^{n-a_2}} \\ &= \frac{[p_{21}]^{\frac{n+S_n}{2}} [p_{11}]^{\frac{n-S_n}{2}} [q_{21}]^{\frac{n+\bar{S}_n}{2}} [q_{11}]^{\frac{n-\bar{S}_n}{2}}}{[p_{20}]^{\frac{n+S_n}{2}} [p_{10}]^{\frac{n-S_n}{2}} [q_{20}]^{\frac{n+\bar{S}_n}{2}} [q_{10}]^{\frac{n-\bar{S}_n}{2}}} \end{aligned} \tag{3.2.22}$$

Taking logrithm on both sides of the equation (3.2.22),

$$\log l_n = \frac{n+S_n}{2}\log\frac{p_{21}}{p_{20}} + \frac{n-S_n}{2}\log\frac{p_{11}}{p_{10}} + \frac{n+\bar{S}_n}{2}\log\frac{q_{21}}{q_{20}} + \frac{n-\bar{S}_n}{2}\log\frac{q_{11}}{q_{10}}$$

And using the definition of expectation and Wald's identity,

$$E_i\{\log l_n\} = \frac{1}{2}[(E_i n + E_i n E_i x_1)\log\frac{p_{21}}{p_{20}} + (E_i n - E_i n E_i x_1)\log\frac{p_{11}}{p_{10}}$$

$$+ (E_i n + E_i n E_i y_1)\log\frac{q_{21}}{q_{20}} + (E_i n - E_i n E_i y_1)\log\frac{q_{11}}{q_{10}} \qquad (3.2.23)$$

Since $E_1 x_1 = 2p_{21} - 1$, $E_1 y_1 = 2q_{21} - 1$, $E_0 x_1 = 2p_{20} - 1$, and $E_0 y_1 = 2q_{20} - 1$, $E_1\{N\}$ and $E_0\{N\}$ can be simplified to exactly the equations (3.2.4) and (3.2.5).

This gives us an interesting thought that there is more than one approach to a problem. In this case, we can think that the two-level quantization system can be considered as a random walk problem.

3.2.2. The G Function

Utilizing the equations derived in the last section, the G function is expressed by simple substitution.

$$G = E_1 N + E_0 N$$

$$= E_1^{-1}\{V_1\}\frac{aA'(B'-1) + bB'(1-A')}{B'-A'} + E_0^{-1}\{V_1\}\frac{a(B'-1) + b(1-A')}{B'-A'} \qquad (3.3.1)$$

where A' and B' are defined in chapter 2, and $E_1\{V_1\}$ and $E_0\{V_1\}$ are shown in (3.2.4) and (3.2.5).

3.3. Minimization of G Under the Double Exponential Assumption

By applying the double exponential distribution to the G function, one can show how the system performs numerically and what the optimal system will be. The reason that a double exponential distribution function is chosen is that it is easy to

manipulate.

3.3.1. Probability Density Function

For sensor 1:

$$H_1: \quad f_1(X) = \frac{\lambda_1}{2} e^{-\lambda_1 |X-\mu_{12}|} \tag{3.5.1}$$

$$H_0: \quad f_0(X) = \frac{\lambda_1}{2} e^{-\lambda_1 |X-\mu_{11}|} \tag{3.5.2}$$

For sensor 2:

$$H_1: \quad f_1(Y) = \frac{\lambda_2}{2} e^{-\lambda_2 |Y-\mu_{22}|} \tag{3.5.3}$$

$$H_0: \quad f_0(Y) = \frac{\lambda_2}{2} e^{-\lambda_2 |Y-\mu_{21}|} \tag{3.5.4}$$

3.3.2. The Sequential Probability Ratio Test

The log LR is defined in (2.3.4). By applying it to the two-level quantizer case,

$$\begin{aligned} \log l_n &= \sum_{k=1}^{n} \log \frac{f_1(x_k) f_1(y_k)}{f_0(x_k) f_0(y_k)} \\ &= \sum_{k=1}^{n} \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}} \end{aligned} \tag{3.5.5}$$

where $j, l = 1, 2$.

Now, the two random sequence $x_1, x_2, \ldots$ and $y_1, y_2, \ldots$ are sampled sequentially until the random time N where N is the first n that $\log l_n$ is not between a and b. At the time N,

$$\text{Accept } H_1 \qquad \text{if } \log l_n \geq b,$$

$$\text{Accept } H_0 \qquad \text{if } \log l_n \le a.$$

Since α and β are given, and they are defined in (2.3.6) and (2.3.9), then

$$a = \log A'$$

$$= \log \frac{\beta}{1-\alpha} \tag{3.5.6}$$

$$b = \log B'$$

$$= \log \frac{1-\beta}{\alpha}. \tag{3.5.7}$$

3.3.3. Calculation on G function

In order to calculate the function G, it is easier to evaluate $E_1\{N\}$ and $E_0\{N\}$ first. From (3.2.4) and (3.2.5), $E_1\{N\}$ and $E_1\{N\}$ are derived. By applying the double exponential functions,

$$E_1\{N\} = \frac{aA'(B'-1)+b(B'-A')}{B'-A'}\left[p_{21}\log\frac{p_{20}p_{11}}{p_{21}p_{10}} + q_{21}\log\frac{q_{20}q_{11}}{q_{21}q_{10}} + \log\frac{p_{10}q_{10}}{p_{11}q_{11}}\right]^{-1}$$

$$= \frac{aA'(B'-1)+b(B'-A')}{B'-A'}\left[\int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\log\frac{\int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}}{\int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}}\right.$$

$$+\int_{T_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\int_{T_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$\left.+\log\frac{\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}{\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}\right]^{-1} \tag{3.2.8}$$

$$E_0\{N\} = \frac{a(B'-1)+b(1-A')}{B'-A'}\left[p_{20}\log\frac{p_{20}p_{11}}{p_{21}p_{10}} + q_{20}\log\frac{q_{20}q_{11}}{q_{21}q_{10}} + \log\frac{p_{10}q_{10}}{p_{11}q_{11}}\right]^{-1}$$

$$= \frac{a(B'-1)+b(1-A')}{B'-A'}\left[\int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\log\frac{\int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}}{\int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}}\right.$$

$$+\int_{T_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$\left.+\log\frac{\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}{\int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}\right]^{-1} \tag{3.2.9}$$

Then $G = E_1\{N\} + E_0\{N\}$ can be calculated. Below are the list of calculated probabilities of all quantized levels for sensor 1:

If $T_1 > \mu_{12}$,

$$p_{21} = \int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|} = \frac{1}{2}e^{-\lambda_1(T_1-\mu_{12})}$$

$$p_{11} = \int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|} = 1-\frac{1}{2}e^{-\lambda_1(T_1-\mu_{12})}$$

$$p_{20} = \int_{T_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|} = \frac{1}{2}e^{-\lambda_1(T_1-\mu_{11})}$$

$$p_{10} = \int_{-\infty}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|} = 1-\frac{1}{2}e^{-\lambda_1(T_1-\mu_{11})}$$

If $\mu_{11} < T_1 \le \mu_{12}$,

$$p_{21} = 1 - \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{12})}$$

$$p_{11} = \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{12})}$$

$$p_{20} = \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{11})}$$

$$p_{10} = 1 - \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{11})}$$

If $T_1 \le \mu_{11}$,

$$p_{21} = 1 - \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{12})}$$

$$p_{11} = \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{12})}$$

$$p_{20} = 1 - \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{11})}$$

$$p_{10} = \frac{1}{2}e^{-\lambda_1(T_1 - \mu_{11})}$$

For sensor 2, the probabilities of quantized level are similar to those in sensor 1.

If $T_2 > \mu_{22}$,

$$q_{21} = \int_{T_2}^{\infty} \frac{\lambda_2}{2}e^{-\lambda_2 | y - \mu_{22} |} = \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{22})}$$

$$q_{11} = \int_{-\infty}^{T_2} \frac{\lambda_2}{2}e^{-\lambda_2 | y - \mu_{22} |} = 1 - \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{22})}$$

$$q_{20} = \int_{T_2}^{\infty} \frac{\lambda_2}{2}e^{-\lambda_2 | y - \mu_{21} |} = \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{21})}$$

$$q_{10} = \int_{-\infty}^{T_2} \frac{\lambda_2}{2}e^{-\lambda_2 | y - \mu_{21} |} = 1 - \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{21})}$$

If $\mu_{21} < T_2 \le \mu_{22}$,

$$q_{21} = 1 - \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{22})}$$

$$q_{11} = \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{22})}$$

$$q_{20} = \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{21})}$$

$$q_{10} = 1 - \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{21})}$$

If $T_2 \le \mu_{21}$,

$$q_{21} = 1 - \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{22})}$$

$$q_{11} = \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{22})}$$

$$q_{20} = 1 - \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{21})}$$

$$q_{10} = \frac{1}{2}e^{-\lambda_2(T_2 - \mu_{21})}$$

3.4. Numerical Evaluation of G

The optimal sequential system that is studied here is defined as having the minimum amount of data needed to determine whether the target is present under the hypotheses H_1 and H_0. Since G is the sum of the conditional expectation of the total number of observation N, the expected N under both hypotheses will be minimized if the minimum of G is found. Thus the purpose of the numerical evaluation is to find the best quantizers for both sensors which minimizes the G function.

During the evaluation, different parameters such as μ_{12}, μ_{11}, μ_{21}, μ_{22}, α, and β are used to determine the effect on the system. The values of λ_1 and λ_2 remain the same throughout the study, and $\lambda_1 = 0.25$ and $\lambda_2 = 0.125$. By varing T_1 and T_2, the minimum G is found.

Figure 3.2 shows the G function with respect to T_2 and T_1. The means used here are symmetric about the y-axis, and $\mu_{11} = -1$, $\mu_{12} = 1$, $\mu_{21} = -2$, and $\mu_{22} = 2$. The error probabilities $\alpha = \beta = 0.01$.

As it shown, the G function is at its minimum when the both T_1 and T_2 of the quantizers are equal to zero.

If the means of the density functions is kept the same, and the error probabilities increase such that $\alpha = \beta = 0.05$, the G is shown in Figure 3.3. The plot in figure 3.4 gives the relation between G, T_1, and T_2 with $\alpha = 0.01$ and $\beta = 0.05$, while the means are still as same as in the previouse figures.

In Figure 3.5, the characteristic of G with means that are not symmetrical about the y-axis is studied. The means of the density functions are $\mu_{11} = -2, \mu_{12} = 1, \mu_{21} = -3, \mu_{22} = 2$, and the error probabilities $\alpha = 0.01$ and $\beta = 0.05$. From this figure, it is clear that the optimal quantizer has its quantized level selected such that it is in the center of the mean.

Figure 3.6 gives another perspective when the $\mu_{11} = -3, \mu_{12} = 3$, $\mu_{21} = -5, \mu_{22} = 5, \alpha = 0.05$, and $\beta = 0.01$. The optimal quantized level is not in the center of means, $T_1 = -0.1$, $T_2 = -0.2$. Again in Figure 3.7 shows the same conclusion with a different set of error probabilities, $\alpha = 0.01$, $\beta = 0.05$. The G is minimum when $T_1 = 0.3$ and $T_2 = 0.1$.

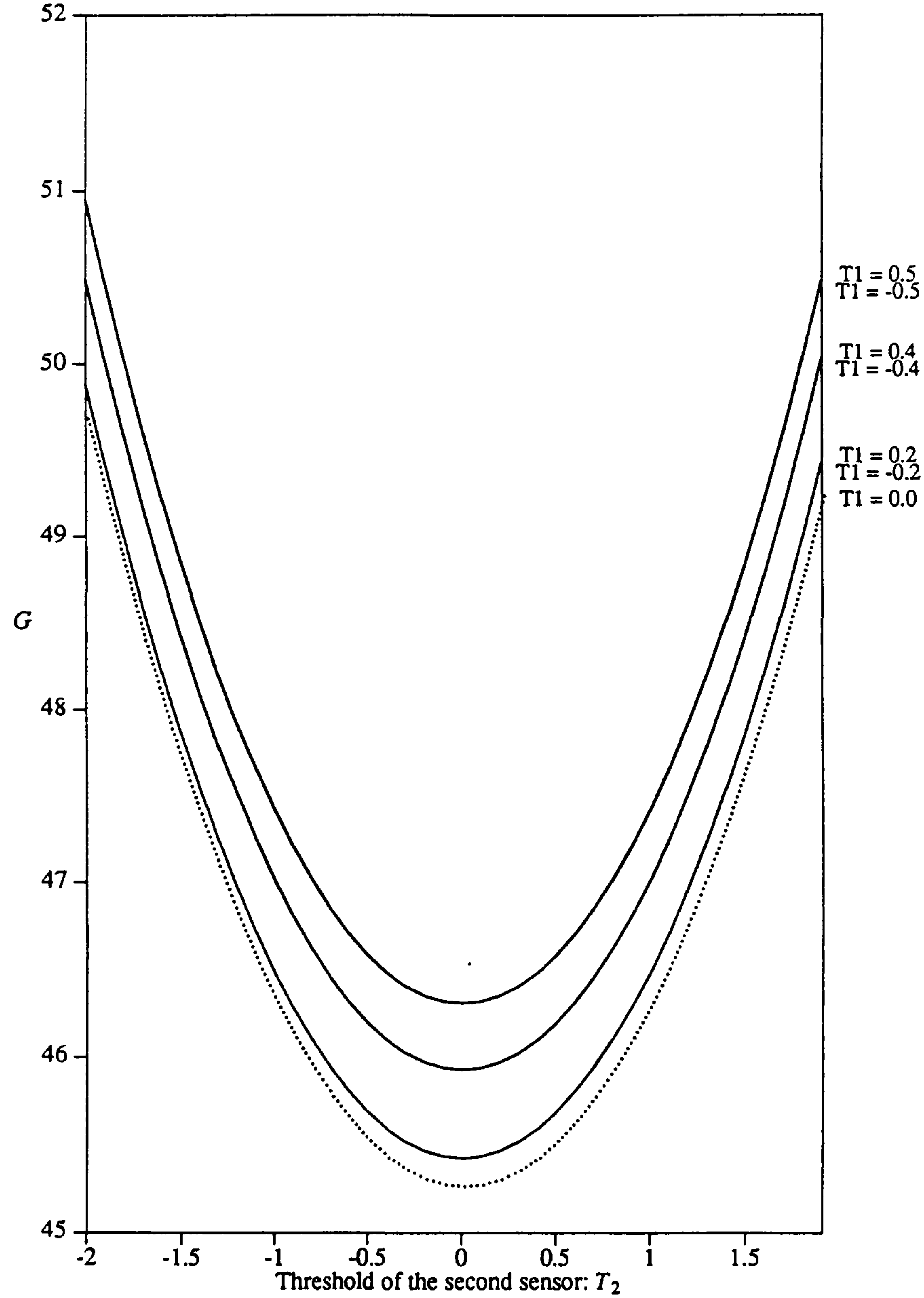

Figure 3.2 G with $\alpha = \beta = 0.01$

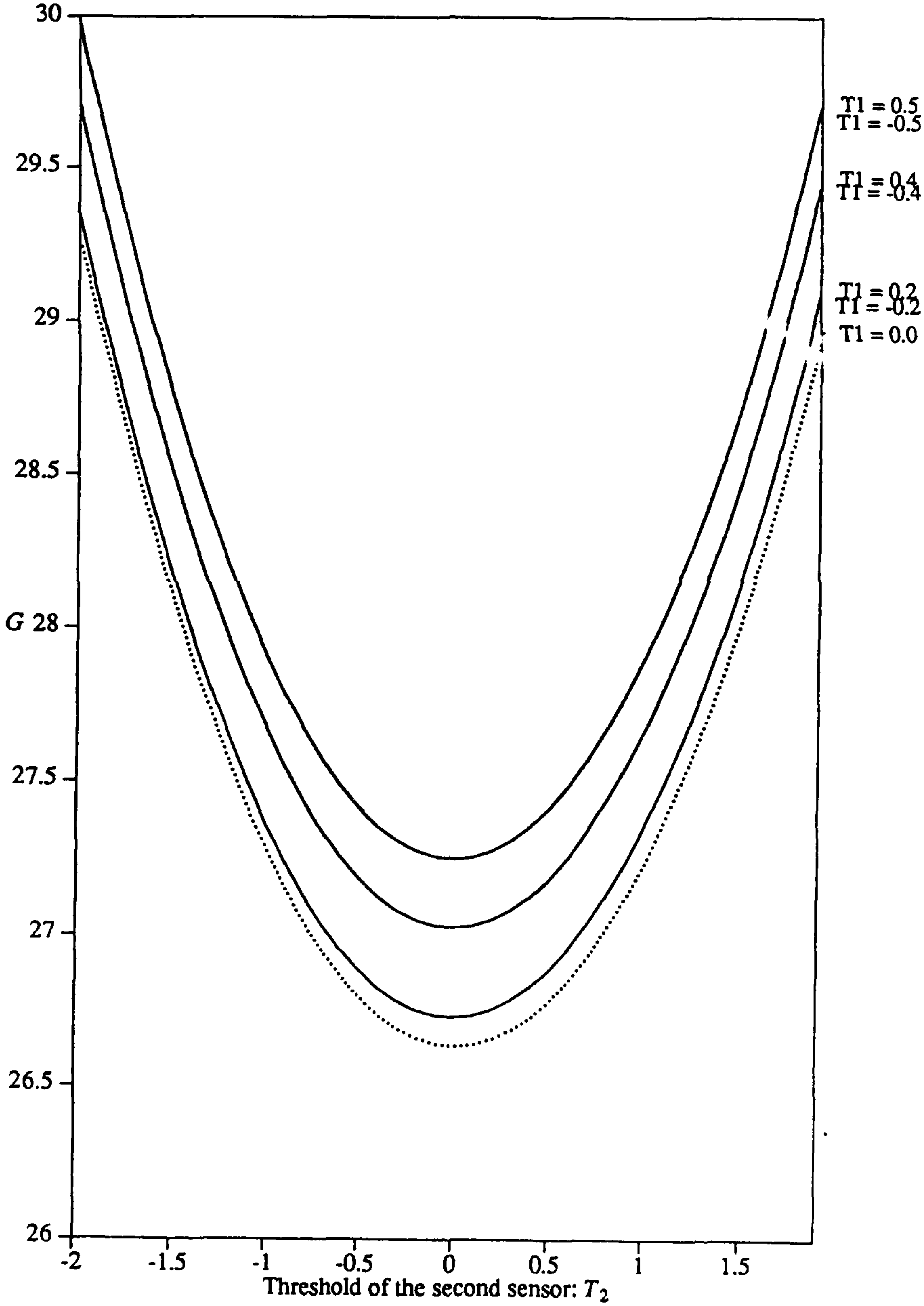

Figure 3.3 G with $\alpha = \beta = 0.05$

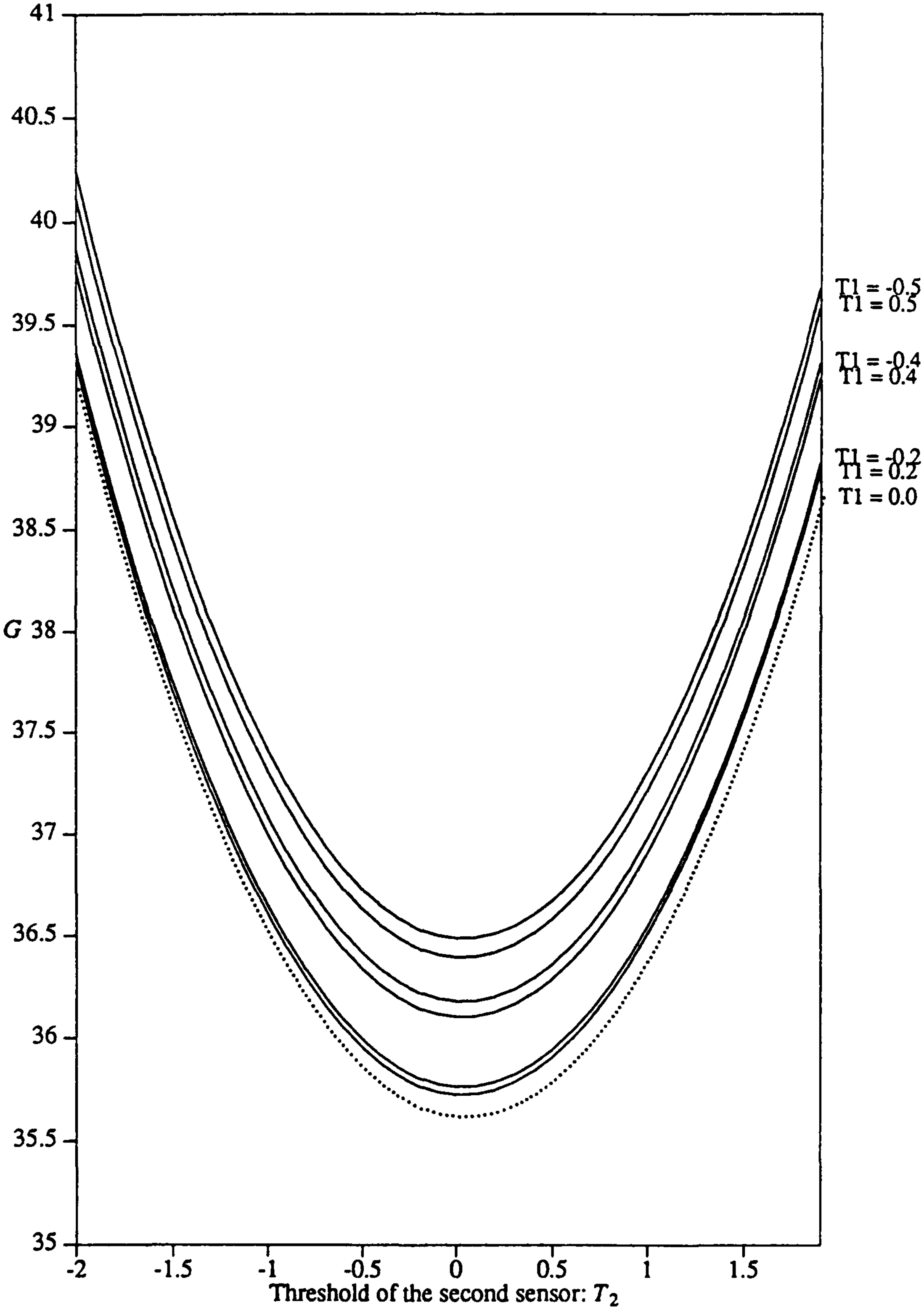

Figure 3.4 G with $\alpha = .01, \beta = .05$

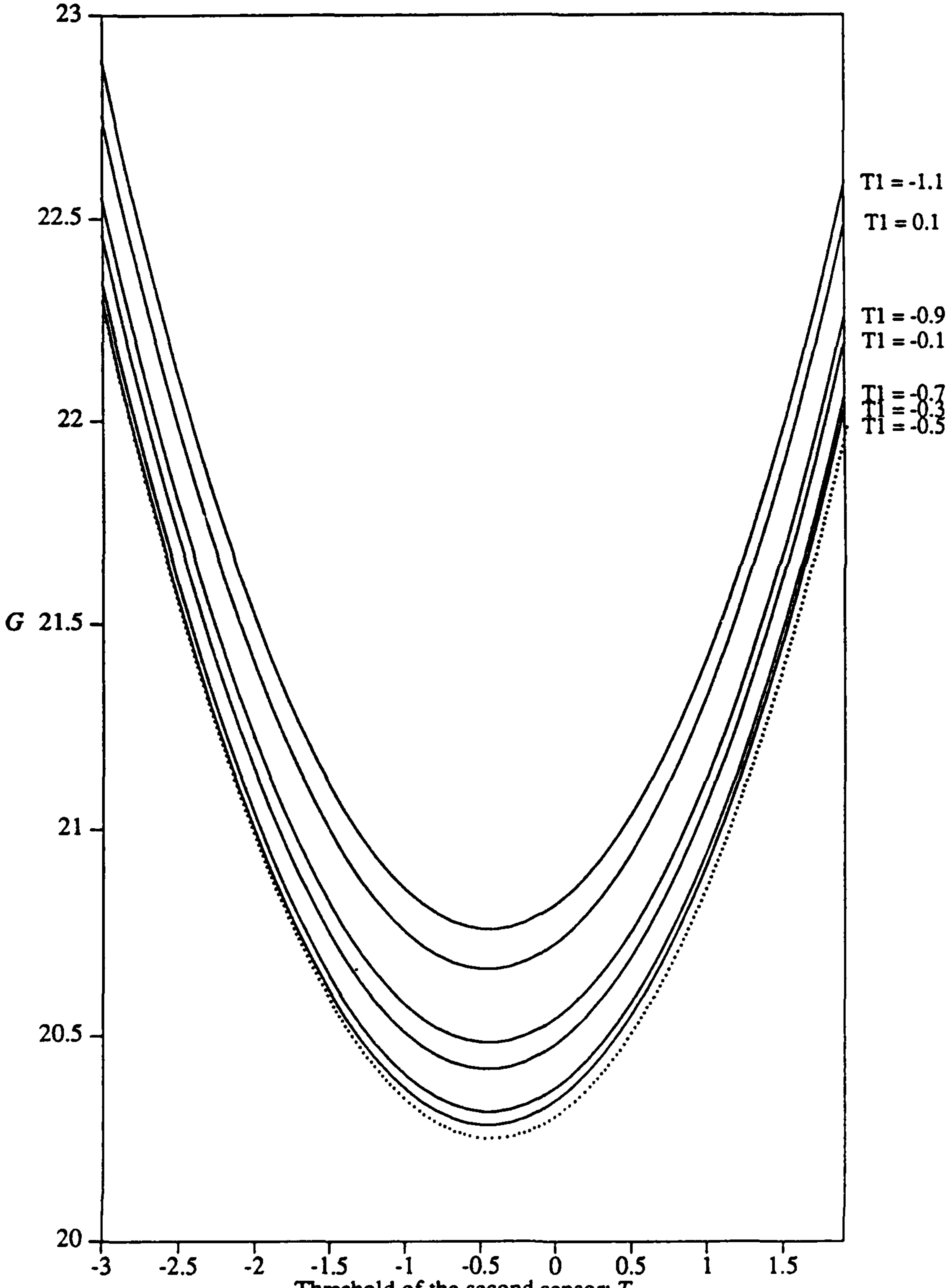

Figure 3.5 G with $\alpha = .01, \beta = .05$ and means at -2,1 and -3,2

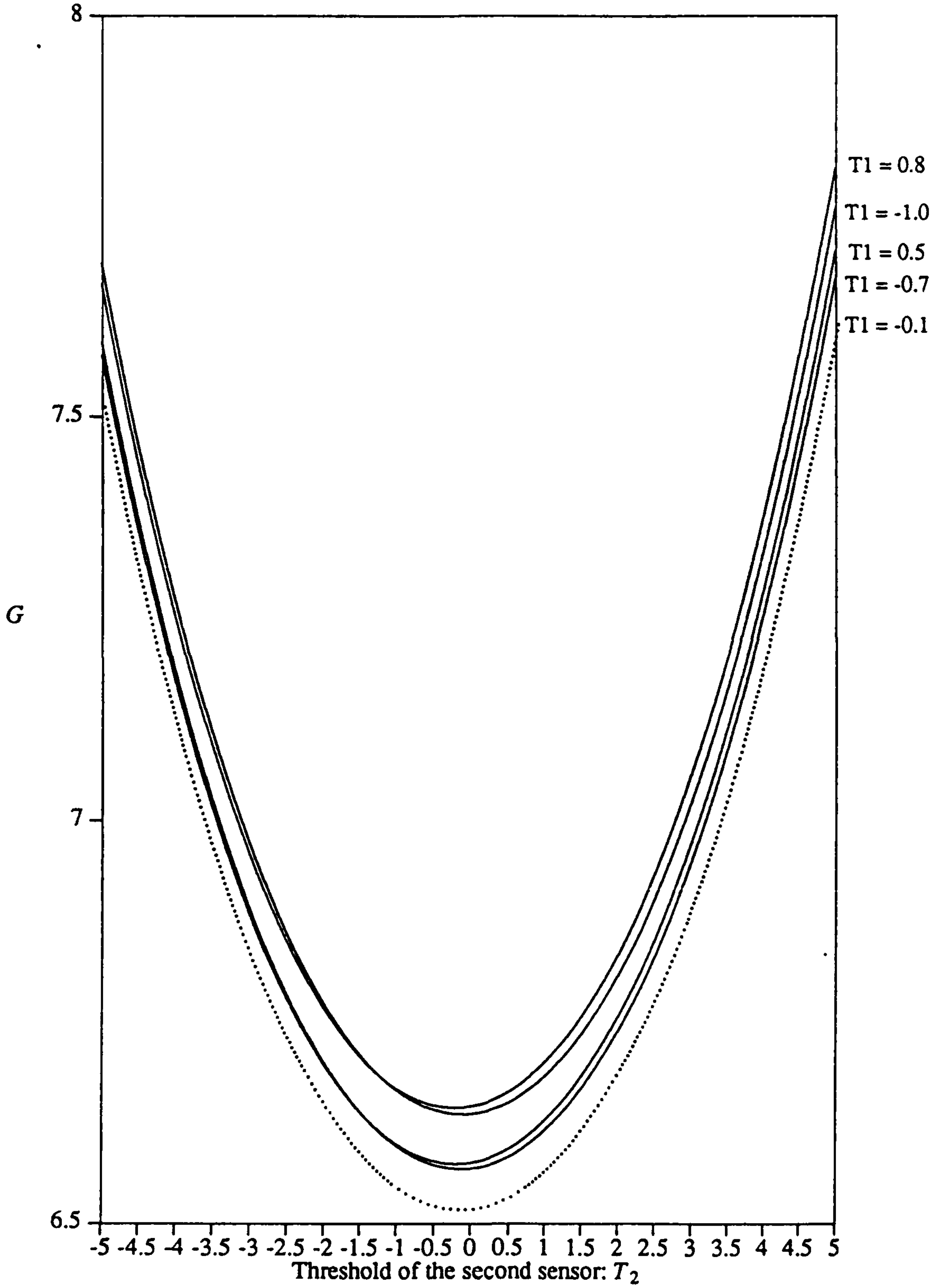

Figure 3.6 G with $\alpha = .05, \beta = .01$ and means at -3,3 and -5,5

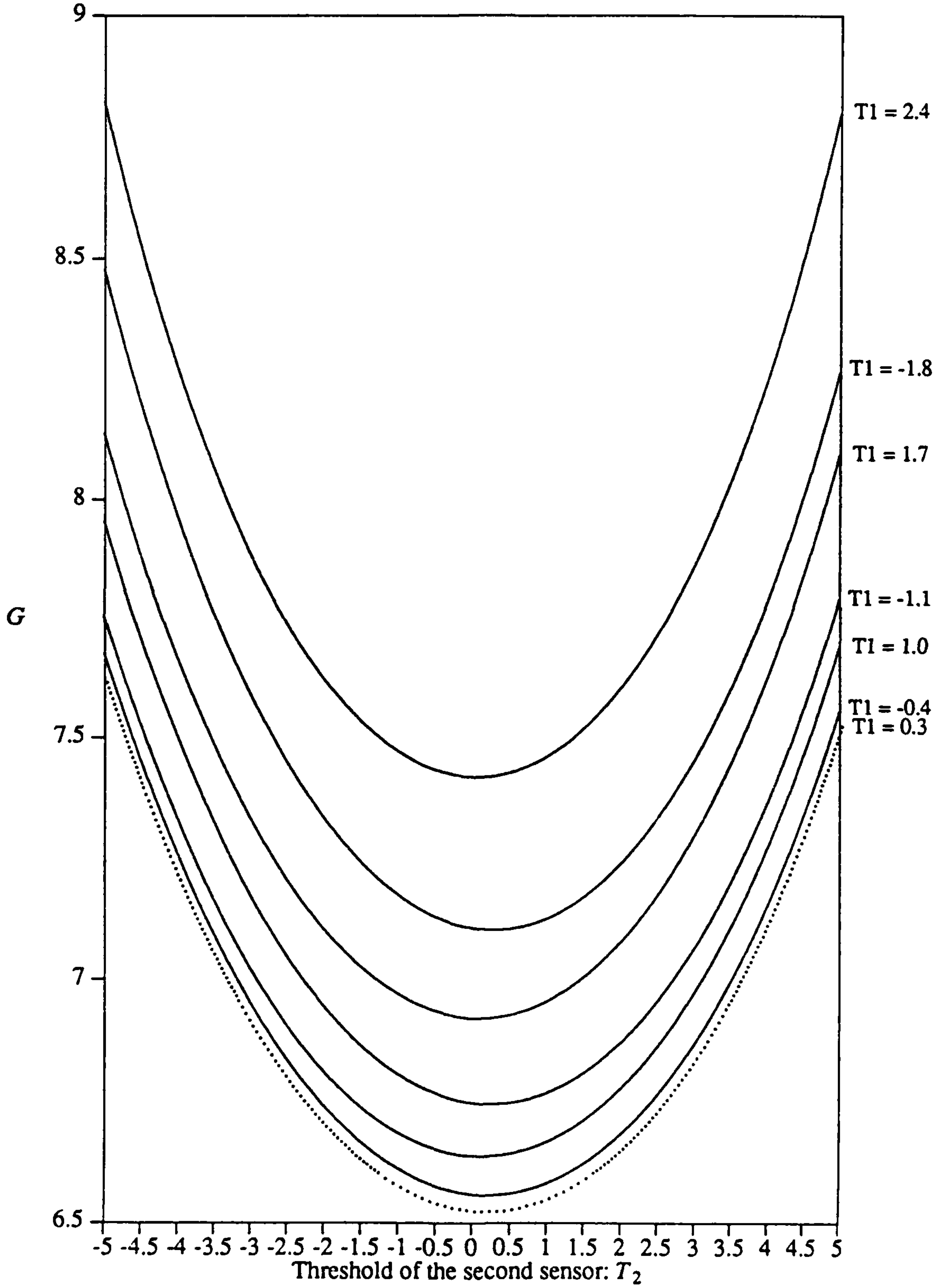

Figure 3.7 G with $\alpha = .01, \beta = .05$ and means at -3,3 and -5,5

CHAPTER 4

Analysis of a Two-Sensor System with Four-level Quantizer

4.1. The Model and Configuration

The process of this sequential detection system is the same as in chapter 2 and chapter 3. The two independent sensors send quantized observations to the fusion center which employs a sequential detection scheme to determine whether a signal is present or not.

The only difference is that the sensors use four-level quantizers instead of two-level ones. The expected number of data will be reduced, since the data from the four-level quantizer contain more information than the data from the two-level quantizer.

4.1.1. The Model of Sensor

As shown in Figure 4.1, each quantizer consists of two variables, namely T_i and D_i where $i = 1,2$. It divides the probability space into four regions. These are $(-\infty, T_i - D_i]$, $(T_i - D_i, T_i]$, $(T_i, T_i + D_i]$, and $(T_i + D_i, +\infty)$. After the sensor receives a signal, depending on which region it falls in, the received data is quantized to 1, 2, 3, or 4. Then this quantized value is sent to the fusion center for processing.

Below are the list of probabilities associated with each quantized level:

For sensor 1:

Four-level Quantizer System

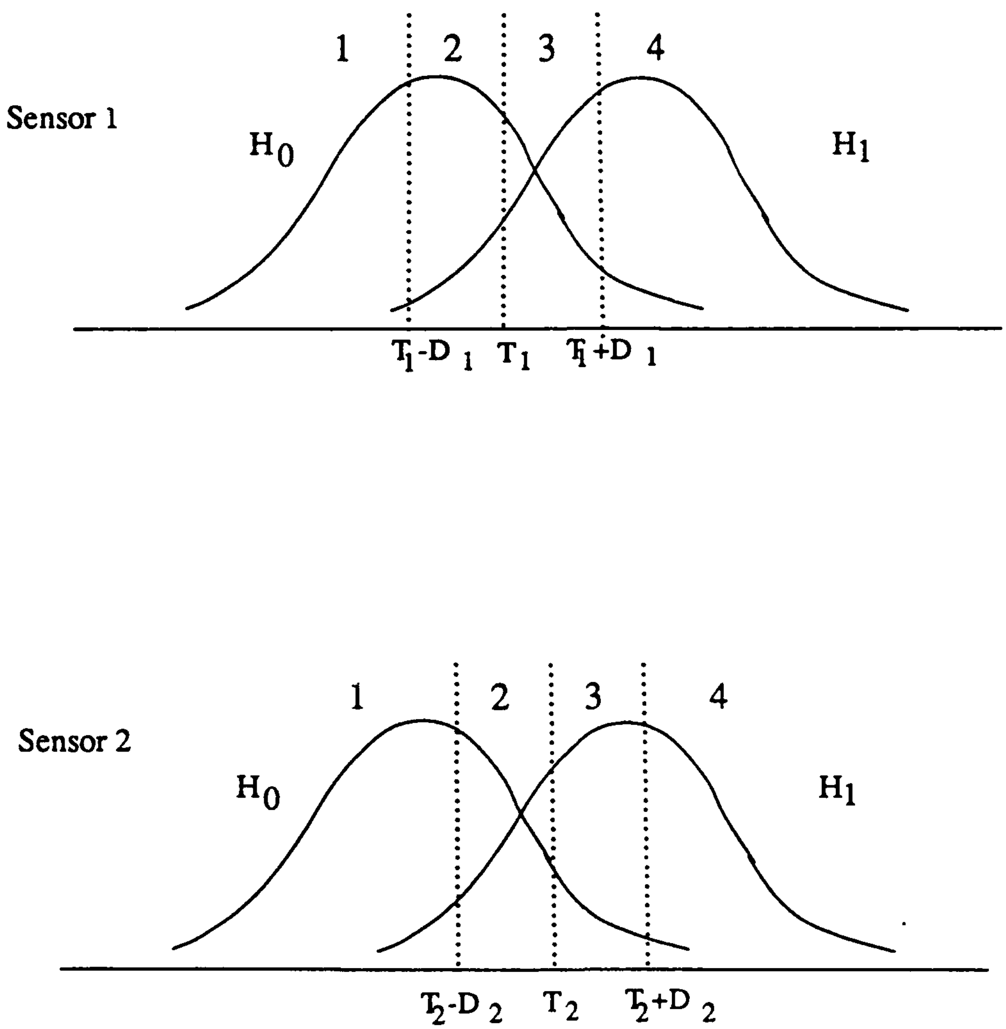

Figure 4.1 Four-level Quantized Probability Space

$$p_{41} = P(X_1 \in (T_1 + D_1, +\infty) \mid H_1) = P(x_1 = 4 \mid H_1) \tag{4.1.1}$$

$$p_{31} = P(X_1 \in (T_1, T_1 + D_1] \mid H_1) = P(x_1 = 3 \mid H_1) \tag{4.1.2}$$

$$p_{21} = P(X_1 \in (T_1 - D_1, T_1] \mid H_1) = P(x_1 = 2 \mid H_1) \tag{4.1.3}$$

$$p_{11} = P(X_1 \in (-\infty, T_1 - D_1] \mid H_1) = P(x_1 = 1 \mid H_1) \tag{4.1.4}$$

$$p_{40} = P(X_1 \in (T_1 + D_1, +\infty) \mid H_0) = P(x_1 = 4 \mid H_0) \tag{4.1.5}$$

$$p_{30} = P(X_1 \in (T_1, T_1 + D_1] \mid H_0) = P(x_1 = 3 \mid H_0) \tag{4.1.6}$$

$$p_{20} = P(X_1 \in (T_1 - D_1, T_1] \mid H_0) = P(x_1 = 2 \mid H_0) \tag{4.1.7}$$

$$p_{10} = P(X_1 \in (-\infty, T_1 - D_1] \mid H_0) = P(x_1 = 1 \mid H_0) \tag{4.1.8}$$

For sensor 2:

$$q_{41} = P(Y_1 \in (T_1 + D_2, +\infty) \mid H_1) = P(x_1 = 4 \mid H_1) \tag{4.1.9}$$

$$q_{31} = P(Y_1 \in (T_1, T_1 + D_2] \mid H_1) = P(x_1 = 3 \mid H_1) \tag{4.1.10}$$

$$q_{21} = P(Y_1 \in (T_1 - D_2, T_1] \mid H_1) = P(x_1 = 2 \mid H_1) \tag{4.1.11}$$

$$q_{11} = P(Y_1 \in (-\infty, T_1 - D_2] \mid H_1) = P(x_1 = 1 \mid H_1) \tag{4.1.12}$$

$$q_{40} = P(Y_1 \in (T_1 + D_2, +\infty) \mid H_0) = P(y_1 = 4 \mid H_0) \tag{4.1.13}$$

$$q_{30} = P(Y_1 \in (T_1, T_1 + D_2] \mid H_0) = P(y_1 = 3 \mid H_0) \tag{4.1.14}$$

$$q_{20} = P(Y_1 \in (T_1 - D_2, T_1] \mid H_0) = P(y_1 = 2 \mid H_0) \tag{4.1.15}$$

$$q_{10} = P(Y_1 \in (-\infty, T_1 - D_2] \mid H_0) = P(y_1 = 1 \mid H_0) \tag{4.1.16}$$

4.2. Evaluate the Expected Value of N ($E_i\{N\}$)

Using the general expression of $E_i\{V_1\}$ in (2.5.6), one can derive $E_i\{V_1\}$ for the four-level quantizer system.

$$E_i\{V_1\} = \sum_{j=1}^{4}\sum_{l=i}^{4} p_{ji} q_{li} \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}}$$

$$= p_{1i} q_{1i} \log \frac{p_{11} q_{11}}{p_{10} q_{10}} + p_{1i} q_{2i} \log \frac{p_{11} q_{21}}{p_{10} q_{20}} + p_{1i} q_{3i} \log \frac{p_{11} q_{31}}{p_{10} q_{30}}$$

$$+ p_{1i}q_{4i}\log\frac{p_{11}q_{41}}{p_{10}q_{40}} + p_{2i}q_{1i}\log\frac{p_{21}q_{11}}{p_{20}q_{20}} + p_{2i}q_{2i}\log\frac{p_{21}q_{21}}{p_{20}q_{20}}$$

$$+ p_{2i}q_{3i}\log\frac{p_{21}q_{31}}{p_{20}q_{30}} + p_{2i}q_{4i}\log\frac{p_{21}q_{41}}{p_{20}q_{40}} + p_{3i}q_{1i}\log\frac{p_{31}q_{11}}{p_{30}q_{10}}$$

$$+ p_{3i}q_{2i}\log\frac{p_{31}q_{21}}{p_{30}q_{20}} + p_{3i}q_{3i}\log\frac{p_{31}q_{31}}{p_{30}q_{30}} + p_{3i}q_{4i}\log\frac{p_{31}q_{41}}{p_{30}q_{40}}$$

$$+ p_{4i}q_{1i}\log\frac{p_{41}q_{11}}{p_{40}q_{20}} + p_{4i}q_{2i}\log\frac{p_{41}q_{21}}{p_{40}q_{20}} + p_{4i}q_{3i}\log\frac{p_{41}q_{31}}{p_{40}q_{30}}$$

$$+ p_{4i}q_{4i}\log\frac{p_{41}q_{41}}{p_{40}q_{40}} \qquad (4.2.1)$$

Then the $E_i\{N\}$ can be expressed as follows:

$$E_1\{N\} = \frac{aA'(B'-1)+b(B'-A')}{B'-A'}\left[\sum_{j=1}^{4}\sum_{l=1}^{4} p_{j1}q_{l1}\log\frac{p_{j1}q_{l1}}{p_{j0}q_{l0}}\right]^{-1} \qquad (4.2.2)$$

$$E_0\{N\} = \frac{a(B'-1)+b(1-A')}{B'-A'}\left[\sum_{j=1}^{4}\sum_{l=1}^{4} p_{j0}q_{l0}\log\frac{p_{j1}q_{l1}}{p_{j0}q_{l0}}\right]^{-1} \qquad (4.2.3)$$

where a, b, A', and B' are all defined in chapter 2.

4.2.1. The G Function

In chapter 2, the G function is defined in (2.6.1). By appling the functions $E_1\{N\}$ and $E_0\{N\}$ derived in the last section, G becomes

$$\begin{aligned} G &= E_1 N + E_0 N \\ &= E_1^{-1}\{V_1\}\frac{aA'(B'-1)+bB'(1-A')}{B'-A'} + E_0^{-1}\{V_1\}\frac{a(B'-1)+b(1-A')}{B'-A'} \\ &= \frac{aA'(B'-1)+b(B'-A')}{B'-A'}\left[\sum_{j=1}^{4}\sum_{l=1}^{4} p_{j1}q_{l1}\log\frac{p_{j1}q_{l1}}{p_{j0}q_{l0}}\right]^{-1} \\ &\quad + \frac{a(B'-1)+b(1-A')}{B'-A'}\left[\sum_{j=1}^{4}\sum_{l=1}^{4} p_{j0}q_{l0}\log\frac{p_{j1}q_{l1}}{p_{j0}q_{l0}}\right]^{-1} \end{aligned} \qquad (4.3.1)$$

4.3. Minimization of G Under the Double Exponential Assumption

4.3.1. Probability Density Function

Double exponential density functions are applied to the equation G to find the optimal system. We assume that the double exponential probability density functions are as same as in chapter 3.

For sensor 1:

$$H_1: \quad f_1(X) = \frac{\lambda_1}{2} e^{-\lambda_1 |X - \mu_{12}|} \tag{4.4.1}$$

$$H_0: \quad f_0(X) = \frac{\lambda_1}{2} e^{-\lambda_1 |X - \mu_{11}|} \tag{4.4.2}$$

For sensor 2:

$$H_1: \quad f_1(Y) = \frac{\lambda_2}{2} e^{-\lambda_2 |Y - \mu_{22}|} \tag{4.4.3}$$

$$H_0: \quad f_0(Y) = \frac{\lambda_2}{2} e^{-\lambda_2 |Y - \mu_{21}|} \tag{4.4.4}$$

4.3.2. The Sequential Probability Ratio Test

For four-level quantizer, the LR is defined as:

$$\begin{aligned} \log l_n &= \sum_{k=1}^{n} \log \frac{f_1(x_k) f_1(y_k)}{f_0(x_k) f_0(y_k)} \\ &= \sum_{k=1}^{n} \log \frac{p_{j1} q_{l1}}{p_{j0} q_{l0}} \end{aligned} \tag{4.4.5}$$

where $j,l = 1,2,3,4$. Then the SPRT can be restated as sampling the two random sequences $x_1, x_2, \ldots$ and $y_1, y_2, \ldots$ sequentially until the random time N, where

N = first $n \geq 1$ such that $\log l_n \notin (a,b)$

$= \infty$ if $\log l_n \in (a,b)$ for all n.

At the time N,

$$\text{Accept } H_1 \quad \text{if } \log l_n \geq b,$$

$$\text{Accept } H_0 \quad \text{if } \log l_n \leq a.$$

4.3.3. Calculation on G function

Equations (4.2.2) and (4.2.3) give the expressions for $E_1\{N\}$ and $E_1\{N\}$. Now using the double exponential functions,

$$E_1\{N\} = \frac{aA'(B'-1)+b(B'-A')}{B'-A'}\left[\sum_{j=1}^{4}\sum_{l=1}^{4} p_{j1}q_{l1}\log\frac{p_{j1}q_{l1}}{p_{j0}q_{l0}}\right]^{-1}$$

$$= \frac{aA'(B'-1)+b(B'-A')}{B'-A'}\left[p_{11}q_{11}\log\frac{p_{11}q_{11}}{p_{10}q_{10}} + p_{11}q_{21}\log\frac{p_{11}q_{21}}{p_{10}q_{20}}\right.$$

$$+ p_{11}q_{31}\log\frac{p_{11}q_{31}}{p_{10}q_{30}} + p_{11}q_{41}\log\frac{p_{11}q_{41}}{p_{10}q_{40}} + p_{21}q_{11}\log\frac{p_{21}q_{11}}{p_{20}q_{10}}$$

$$+ p_{21}q_{21}\log\frac{p_{21}q_{21}}{p_{20}q_{20}} + p_{21}q_{31}\log\frac{p_{21}q_{31}}{p_{20}q_{30}} + p_{21}q_{41}\log\frac{p_{21}q_{41}}{p_{20}q_{40}}$$

$$+ p_{31}q_{11}\log\frac{p_{31}q_{11}}{p_{30}q_{10}} + p_{31}q_{21}\log\frac{p_{31}q_{21}}{p_{30}q_{20}} + p_{31}q_{31}\log\frac{p_{31}q_{31}}{p_{30}q_{30}}$$

$$+ p_{31}q_{41}\log\frac{p_{31}q_{41}}{p_{30}q_{40}} + p_{41}q_{11}\log\frac{p_{41}q_{11}}{p_{40}q_{10}} + p_{41}q_{21}\log\frac{p_{41}q_{21}}{p_{40}q_{20}} \; .$$

$$\left. + p_{41}q_{31}\log\frac{p_{41}q_{31}}{p_{40}q_{30}} + p_{41}q_{41}\log\frac{p_{41}q_{41}}{p_{40}q_{40}}\right]^{-1}$$

$$= \frac{aA'(B'-1)+b(B'-A')}{B'-A'}$$

$$
\left[\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}}{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}\right.
$$

$$
+\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}}
$$

$$
+\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}}{\displaystyle\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$
+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\displaystyle\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}
$$

$$+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$\left.+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}}\right]^{-1} \tag{4.4.6}$$

$$E_0\{N\}=\frac{a(B-1)+b(1-A')}{B'-A'}\left[\sum_{j=1}^{4}\sum_{l=1}^{4}p_{j0}q_{l0}\log\frac{p_{j1}q_{l1}}{p_{j0}q_{l0}}\right]^{-1}$$

$$=\frac{a(B'-1)+b(1-A')}{B'-A'}\left[p_{10}q_{10}\log\frac{p_{11}q_{11}}{p_{10}q_{10}}\right.$$

$$+p_{10}q_{20}\log\frac{p_{11}q_{21}}{p_{10}q_{20}}+p_{10}q_{30}\log\frac{p_{11}q_{31}}{p_{10}q_{30}}+p_{10}q_{40}\log\frac{p_{11}q_{41}}{p_{10}q_{40}}$$

$$+p_{20}q_{10}\log\frac{p_{21}q_{11}}{p_{20}q_{10}}+p_{20}q_{20}\log\frac{p_{21}q_{21}}{p_{20}q_{20}}+p_{20}q_{30}\log\frac{p_{21}q_{31}}{p_{20}q_{30}}$$

$$+p_{20}q_{40}\log\frac{p_{21}q_{41}}{p_{20}q_{40}}+p_{30}q_{10}\log\frac{p_{31}q_{11}}{p_{30}q_{10}}+p_{30}q_{20}\log\frac{p_{31}q_{21}}{p_{30}q_{20}}$$

$$+p_{30}q_{30}\log\frac{p_{31}q_{31}}{p_{30}q_{30}}+p_{30}q_{40}\log\frac{p_{31}q_{41}}{p_{30}q_{40}}+p_{40}q_{10}\log\frac{p_{41}q_{11}}{p_{40}q_{20}}$$

$$\left.+p_{40}q_{20}\log\frac{p_{41}q_{21}}{p_{40}q_{20}}+p_{40}q_{30}\log\frac{p_{41}q_{31}}{p_{40}q_{30}}+p_{40}q_{40}\log\frac{p_{41}q_{41}}{p_{40}q_{40}}\right]^{-1}$$

$$=\frac{aA'(B'-1)+b(B'-A')}{B'-A'}$$

$$\left[\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}\log\frac{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}}{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}\right.$$

$$+\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}}$$

$$+\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}}{\int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{-\infty}^{T_2-D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2-D_2}^{T_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2}^{T_2+D_2}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}}$$

$$+\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{21}|}\log\frac{\displaystyle\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_2}{2}e^{-\lambda_2|y-\mu_{22}|}}{\displaystyle\int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}\int_{T_2+D_2}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|}}\Bigg]^{-1} \tag{4.4.7}$$

Combining equations (4.4.6) and (4.4.7), $G = E_1\{N\} + E_0\{N\}$ can be calculated. Below is the list of the probability of each quantized level under different T_1, D_1, T_2, and D_2 where $\mu_{11} < T_1 < \mu_{12}$ and $\mu_{21} < T_2 < \mu_{22}$. It has been found by our evaluation that G has its minima at the region $\mu_{11} < T_1 < \mu_{12}$ and $\mu_{21} < T_2 < \mu_{22}$.

For sensor 1,

If $T_1 + D_1 \geq \mu_{12}$ and $T_1 - D_1 \geq \mu_{11}$,

$$p_{41} = \int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|} = \frac{1}{2}e^{-\lambda_1(T_1+D_1-\mu_{12})}$$

$$p_{31} = \int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|} = 1 - \frac{1}{2}e^{-\lambda_1(\mu_{12}-T_1)} - \frac{1}{2}e^{-\lambda_1(T_1+D_1-\mu_{12})}$$

$$p_{21} = \int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|} = \frac{1}{2}e^{-\lambda_1(\mu_{12}-T_1)} - \frac{1}{2}e^{-\lambda_1(\mu_{12}-T_1+D_1)}$$

$$p_{11} = \int_{-\infty}^{T_1-D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{12}|} = \frac{1}{2}e^{-\lambda_1(\mu_{12}-T_1+D_1)}$$

$$p_{40} = \int_{T_1+D_1}^{\infty}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|} = \frac{1}{2}e^{-\lambda_1(T_1+D_1-\mu_{11})}$$

$$p_{30} = \int_{T_1}^{T_1+D_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|} = \frac{1}{2}e^{-\lambda_1(T_1-\mu_{11})} - \frac{1}{2}e^{-\lambda_1(T_1+D_1-\mu_{11})}$$

$$p_{20} = \int_{T_1-D_1}^{T_1}\frac{\lambda_1}{2}e^{-\lambda_1|x-\mu_{11}|} = \frac{1}{2}e^{-\lambda_1(T_1-D_1-\mu_{11})} - \frac{1}{2}e^{-\lambda_1(T_1-\mu_{11})}$$

$$p_{10} = \int_{-\infty}^{T_1 - D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = 1 - \frac{1}{2} e^{-\lambda_1 (T_1 - D_1 - \mu_{11})}$$

If $T_1 + D_1 \geq \mu_{12}$ and $T_1 - D_1 < \mu_{11}$

$$p_{41} = \int_{T_1 + D_1}^{\infty} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1 (T_1 + D_1 - \mu_{12})}$$

$$p_{31} = \int_{T_1}^{T_1 + D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = 1 - \frac{1}{2} e^{-\lambda_1 (\mu_{12} - T_1)} - \frac{1}{2} e^{-\lambda_1 (T_1 + D_1 - \mu_{12})}$$

$$p_{21} = \int_{T_1 - D_1}^{T_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1 (\mu_{12} - T_1)} - \frac{1}{2} e^{-\lambda_1 (\mu_{12} - T_1 + D_1)}$$

$$p_{11} = \int_{-\infty}^{T_1 - D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1 (\mu_{12} - T_1 + D_1)}$$

$$p_{40} = \int_{T_1 + D_1}^{\infty} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1 (T_1 + D_1 - \mu_{11})}$$

$$p_{30} = \int_{T_1}^{T_1 + D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1 (T_1 - \mu_{11})} - \frac{1}{2} e^{-\lambda_1 (T_1 + D_1 - \mu_{11})}$$

$$p_{20} = \int_{T_1 - D_1}^{T_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = 1 - \frac{1}{2} e^{-\lambda_1 (\mu_{11} - T_1 + D_1)} - \frac{1}{2} e^{-\lambda_1 (T_1 - \mu_{12})}$$

$$p_{10} = \int_{-\infty}^{T_1 - D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1 (\mu_{11} - T_1 + D_1)}$$

If $T_1 + D_1 < \mu_{12}$ and $T_1 - D_1 \geq \mu_{11}$

$$p_{41} = \int_{T_1 + D_1}^{\infty} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = 1 - \frac{1}{2} e^{-\lambda_1 (\mu_{12} - T_1 - D_1)}$$

$$p_{31} = \int_{T_1}^{T_1 + D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1 (\mu_{12} - T_1 - D_1)} - \frac{1}{2} e^{-\lambda_1 (\mu_{12} - T_1)}$$

$$p_{21} = \int_{T_1 - D_1}^{T_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1)} - \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1 + D_1)}$$

$$p_{11} = \int_{-\infty}^{T_1 - D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1 + D_1)}$$

$$p_{40} = \int_{T_1 + D_1}^{\infty} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1(T_1 + D_1 - \mu_{11})}$$

$$p_{30} = \int_{T_1}^{T_1 + D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1(T_1 - \mu_{11})} - \frac{1}{2} e^{-\lambda_1(T_1 + D_1 - \mu_{11})}$$

$$p_{20} = \int_{T_1 - D_1}^{T_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1(T_1 - D_1 - \mu_{11})} - \frac{1}{2} e^{-\lambda_1(T_1 - \mu_{11})}$$

$$p_{10} = \int_{-\infty}^{T_1 - D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = 1 - \frac{1}{2} e^{-\lambda_1(T_1 - D_1 - \mu_{11})}$$

If $T_1 + D_1 < \mu_{12}$ and $T_1 - D_1 < \mu_{11}$

$$p_{41} = \int_{T_1 + D_1}^{\infty} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = 1 - \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1 - D1)}$$

$$p_{31} = \int_{T_1}^{T_1 + D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1 - D_1)} - \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1)}$$

$$p_{21} = \int_{T_1 - D_1}^{T_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1)} - \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1 + D_1)}$$

$$p_{11} = \int_{-\infty}^{T_1 - D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{12}|} = \frac{1}{2} e^{-\lambda_1(\mu_{12} - T_1 + D_1)}$$

$$p_{40} = \int_{T_1 + D_1}^{\infty} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1(T_1 + D_1 - \mu_{11})}$$

$$p_{30} = \int_{T_1}^{T_1 + D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x-\mu_{11}|} = \frac{1}{2} e^{-\lambda_1(T_1 - \mu_{11})} - \frac{1}{2} e^{-\lambda_1(T_1 + D_1 - \mu_{11})}$$

$$p_{20} = \int_{T_1 - D_1}^{T_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x - \mu_{11}|} = 1 - \frac{1}{2} e^{-\lambda_1(\mu_{11} - T_1 + D_1)} - \frac{1}{2} e^{-\lambda_1(T_1 - \mu_{12})}$$

$$p_{10} = \int_{-\infty}^{T_1 - D_1} \frac{\lambda_1}{2} e^{-\lambda_1 |x - \mu_{11}|} = \frac{1}{2} e^{-\lambda_1(\mu_{11} - T_1 + D_1)}$$

For sensor 2, the probabilities of quantized level are similar to those in sensor 1.

For sensor 2,

If $T_2 + D_2 \geq \mu_{22}$ and $T_2 - D_2 \geq \mu_{21}$,

$$q_{41} = \int_{T_2 + D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{22}|} = \frac{1}{2} e^{-\lambda_2(T_2 + D_2 - \mu_{22})}$$

$$q_{31} = \int_{T_2}^{T_2 + D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{22}|} = 1 - \frac{1}{2} e^{-\lambda_2(\mu_{22} - T_2)} - \frac{1}{2} e^{-\lambda_2(T_2 + D_2 - \mu_{22})}$$

$$q_{21} = \int_{T_2 - D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{22}|} = \frac{1}{2} e^{-\lambda_2(\mu_{22} - T_2)} - \frac{1}{2} e^{-\lambda_2(\mu_{22} - T_2 + D_2)}$$

$$q_{11} = \int_{-\infty}^{T_2 - D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{22}|} = \frac{1}{2} e^{-\lambda_2(\mu_{22} - T_2 + D_2)}$$

$$q_{40} = \int_{T_2 + D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2 + D_2 - \mu_{21})}$$

$$q_{30} = \int_{T_2}^{T_2 + D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2 - \mu_{21})} - \frac{1}{2} e^{-\lambda_2(T_2 + D_2 - \mu_{21})}$$

$$q_{20} = \int_{T_2 - D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2 - D_2 - \mu_{21})} - \frac{1}{2} e^{-\lambda_2(T_2 - \mu_{21})}$$

$$q_{10} = \int_{-\infty}^{T_2 - D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y - \mu_{21}|} = 1 - \frac{1}{2} e^{-\lambda_2(T_2 - D_2 - \mu_{21})}$$

If $T_2 + D_2 \geq \mu_{22}$ and $T_2 - D_2 < \mu_{21}$

$$q_{41} = \int_{T_2+D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2 (T_2 + D_2 - \mu_{22})}$$

$$q_{31} = \int_{T_2}^{T_2+D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = 1 - \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2)} - \frac{1}{2} e^{-\lambda_2 (T_2 + D_2 - \mu_{22})}$$

$$q_{21} = \int_{T_2-D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2)} - \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2 + D_2)}$$

$$q_{11} = \int_{-\infty}^{T_2-D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2 + D_2)}$$

$$q_{40} = \int_{T_2+D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2 (T_2 + D_2 - \mu_{21})}$$

$$q_{30} = \int_{T_2}^{T_2+D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2 (T_2 - \mu_{21})} - \frac{1}{2} e^{-\lambda_2 (T_2 + D_2 - \mu_{21})}$$

$$q_{20} = \int_{T_2-D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = 1 - \frac{1}{2} e^{-\lambda_2 (\mu_{21} - T_2 + D_2)} - \frac{1}{2} e^{-\lambda_2 (T_2 - \mu_{22})}$$

$$q_{10} = \int_{-\infty}^{T_2-D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2 (\mu_{21} - T_2 + D_2)}$$

If $T_2 + D_2 < \mu_{22}$ and $T_2 - D_2 \geq \mu_{21}$

$$q_{41} = \int_{T_2+D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = 1 - \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2 - D2)}$$

$$q_{31} = \int_{T_2}^{T_2+D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2 - D_2)} - \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2)}$$

$$q_{21} = \int_{T_2-D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2)} - \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2 + D_2)}$$

$$q_{11} = \int_{-\infty}^{T_2-D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2 (\mu_{22} - T_2 + D_2)}$$

$$q_{40} = \int_{T_2+D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2+D_2-\mu_{21})}$$

$$q_{30} = \int_{T_2}^{T_2+D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2-\mu_{21})} - \frac{1}{2} e^{-\lambda_2(T_2+D_2-\mu_{21})}$$

$$q_{20} = \int_{T_2-D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2-D_2-\mu_{21})} - \frac{1}{2} e^{-\lambda_2(T_2-\mu_{21})}$$

$$q_{10} = \int_{-\infty}^{T_2-D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = 1 - \frac{1}{2} e^{-\lambda_2(T_2-D_2-\mu_{21})}$$

$$q_{30} = \int_{T_2}^{T_2+D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2-\mu_{21})} - \frac{1}{2} e^{-\lambda_2(T_2+D_2-\mu_{21})}$$

$$q_{20} = \int_{T_2-D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2-D_2-\mu_{21})} - \frac{1}{2} e^{-\lambda_2(T_2-\mu_{21})}$$

$$q_{10} = \int_{-\infty}^{T_2-D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = 1 - \frac{1}{2} e^{-\lambda_2(T_2-D_2-\mu_{21})}$$

If $T_2 + D_2 < \mu_{22}$ and $T_2 - D_2 < \mu_{21}$

$$q_{41} = \int_{T_2+D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = 1 - \frac{1}{2} e^{-\lambda_2(\mu_{22}-T_2-D_2)}$$

$$q_{31} = \int_{T_2+D_2}^{\infty} F1 = \frac{1}{2} e^{-\lambda_2(\mu_{22}-T_2-D_2)} - \frac{1}{2} e^{-\lambda_2(\mu_{22}-T_2)}$$

$$q_{21} = \int_{T_2-D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2(\mu_{22}-T_2)} - \frac{1}{2} e^{-\lambda_2(\mu_{22}-T_2+D_2)}$$

$$q_{11} = \int_{-\infty}^{T_2-D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{22}|} = \frac{1}{2} e^{-\lambda_2(\mu_{22}-T_2+D_2)}$$

$$q_{40} = \int_{T_2+D_2}^{\infty} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2+D_2-\mu_{21})}$$

$$q_{30} = \int_{T_2}^{T_2+D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(T_2-\mu_{21})} - \frac{1}{2} e^{-\lambda_2(T_2+D_2-\mu_{21})}$$

$$q_{20} = \int_{T_2-D_2}^{T_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = 1 - \frac{1}{2} e^{-\lambda_2(\mu_{21}-T_2+D_2)} - \frac{1}{2} e^{-\lambda_2(T_2-\mu_{22})}$$

$$q_{10} = \int_{-\infty}^{T_2-D_2} \frac{\lambda_2}{2} e^{-\lambda_2 |y-\mu_{21}|} = \frac{1}{2} e^{-\lambda_2(\mu_{21}-T_2+D_2)}$$

4.4. Numerical Evaluation of the G Function

The numerical evaluations are done by using the G function defined in the previous section. Since we have to deal with four variables, namely D_1, D_2, T_1, and T_2, it is not very convenient to graph G in terms of these variables. The graphs plotted here are the ones with optimal T_1 and T_2 which means that G is the smallest at that pairs of T_1 and T_2. In Figure 4.2, the curves represent the G of the four-level quantizer sequential system (FQSS) with respect to D_1 and D_2. It is under the conditions that $T_1 = T_2 = 0.0$, $\mu_{11} = -1$, $\mu_{21} = 1$, $\mu_{12} = -2$, $\mu_{22} = 2$, and $\alpha = \beta = 0.01$. In the evaluation, G is also calculated using different pairs of T_1 and T_2, and it is shown in Figure 4.3.

In Figure 4.4, the means are kept the same as Figure 4.2, and both α and β increased to 0.05 with $T_1 = 0.0$ and $T_2 = 0.0$. Next, it shows G function with different error probabilities, $\alpha = 0.01$ and $\beta = 0.05$ with $T_1 = T_2 = 0.0$ in Figure 4.5. With means shifted to the left, $\mu_{11} = -2$, $\mu_{21} = 1$, $\mu_{12} = -3$, and $\mu_{22} = 2$, the G is drawn in Figure 4.6 with the same error probabilities as the previouse graph and T_1 = T_2 = -0.5.

Figure 4.7 shows what happens to G, when the means are $\mu_{11} = -3$, $\mu_{21} = 3$, $\mu_{12} = -5$, and $\mu_{22} = 5$. The error probabilities $\alpha = 0.01$ and $\beta = 0.05$ with $T_1 = -0.1$ and $T_2 = -0.2$.

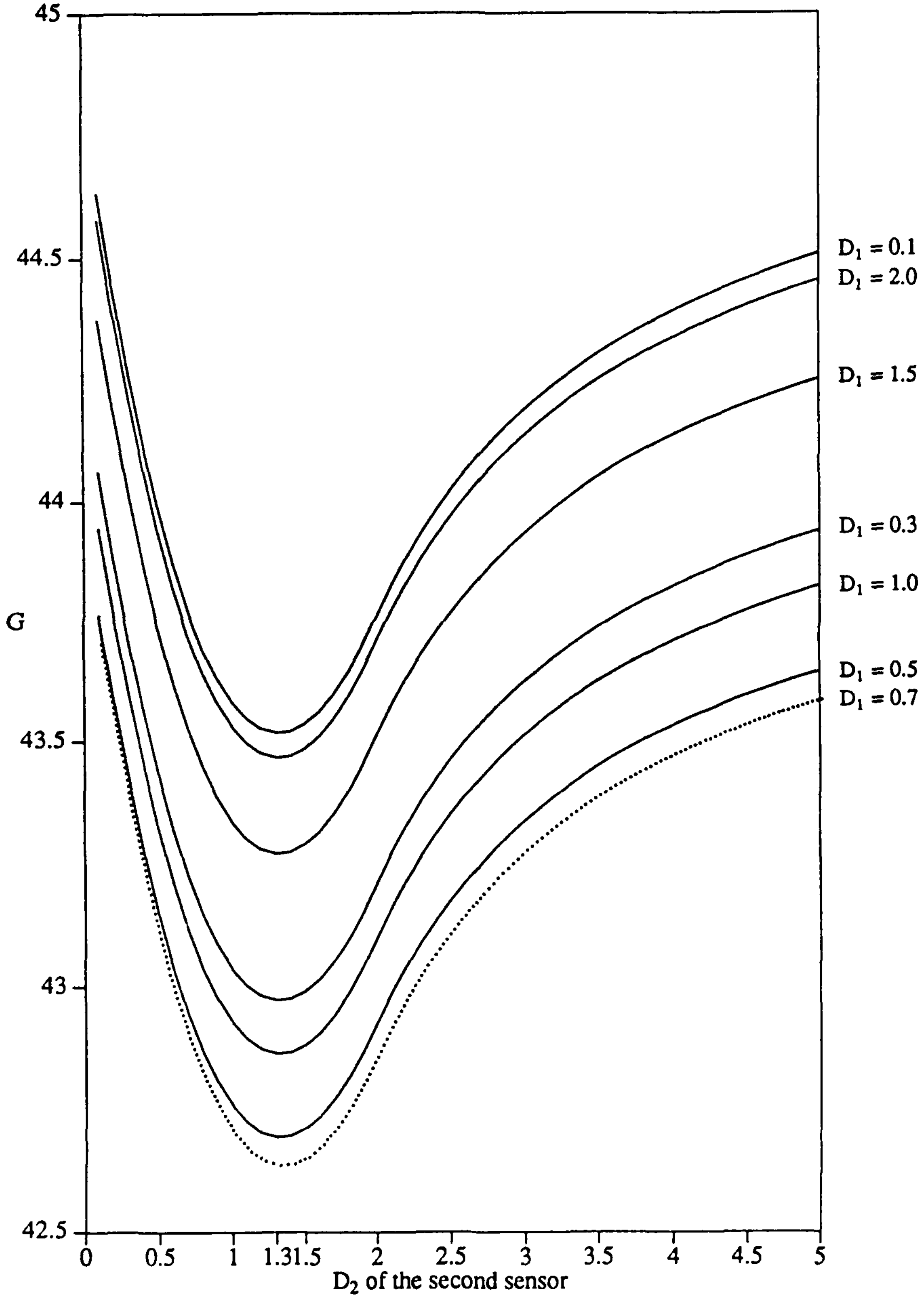

Figure 4.2 G of FQSS with α=β=.01

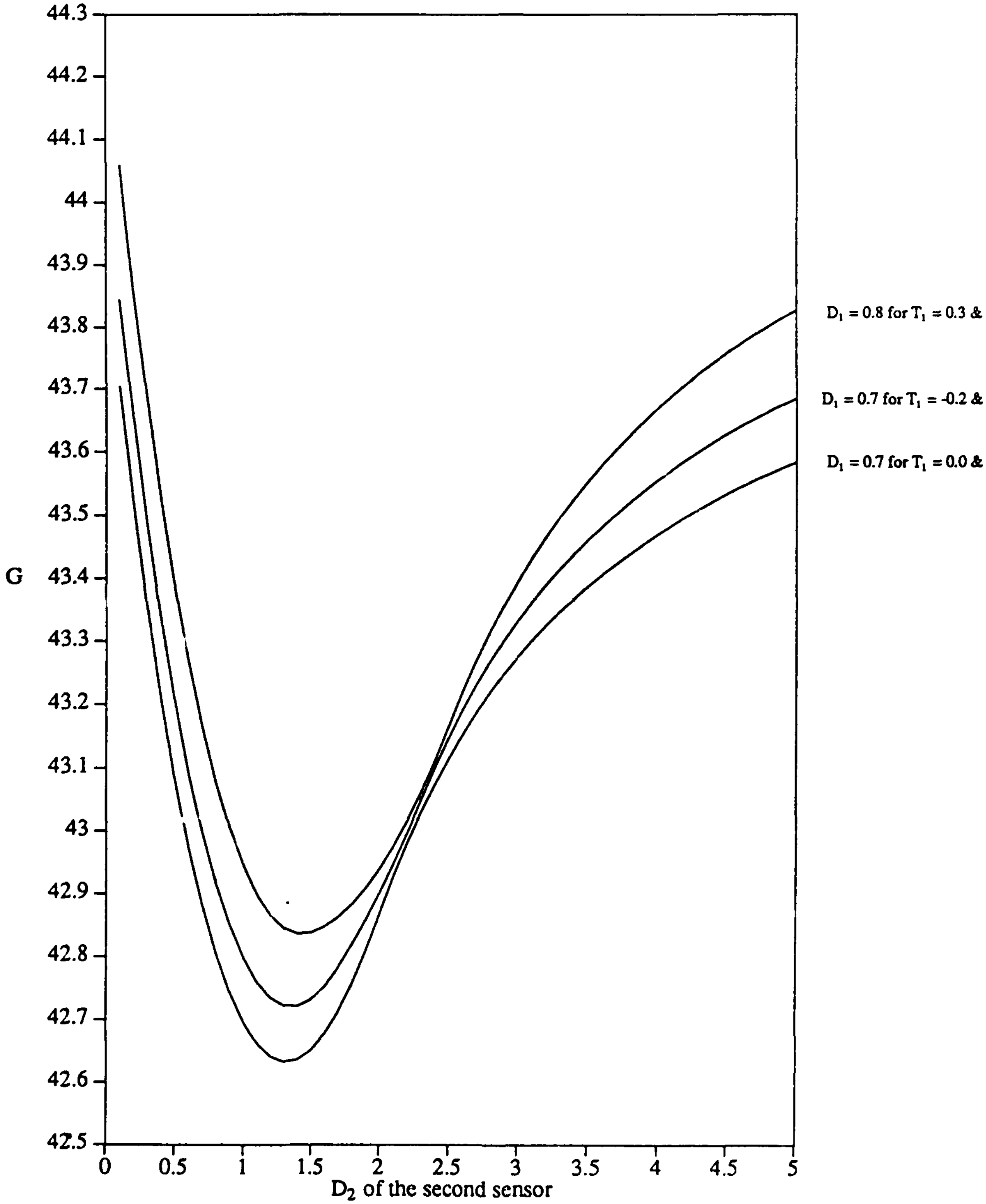

Figure 4.3 G of FQSS with Various T_1 and T_2

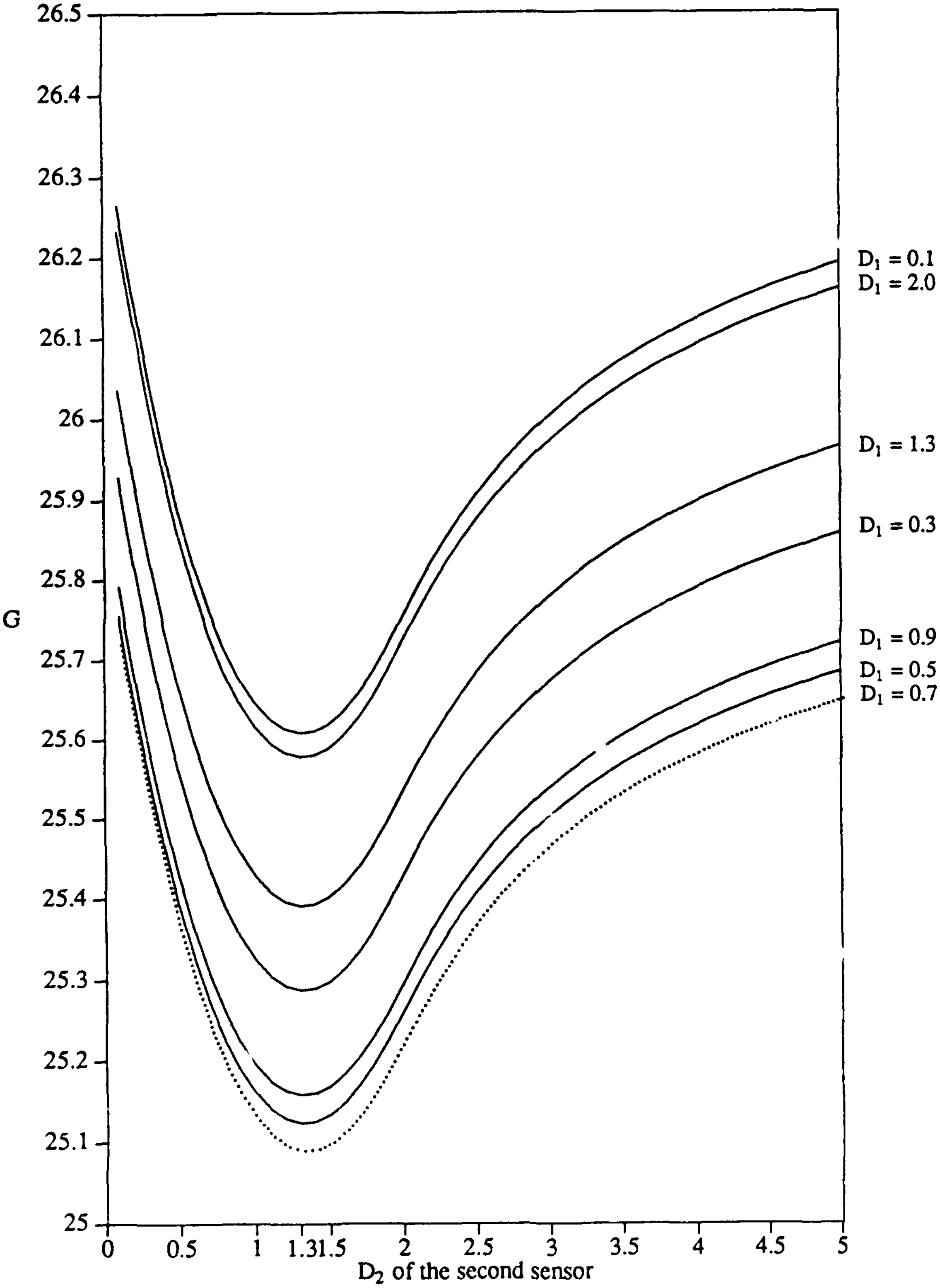

Figure 4.4 G of FQSS with $\alpha=\beta=.05$

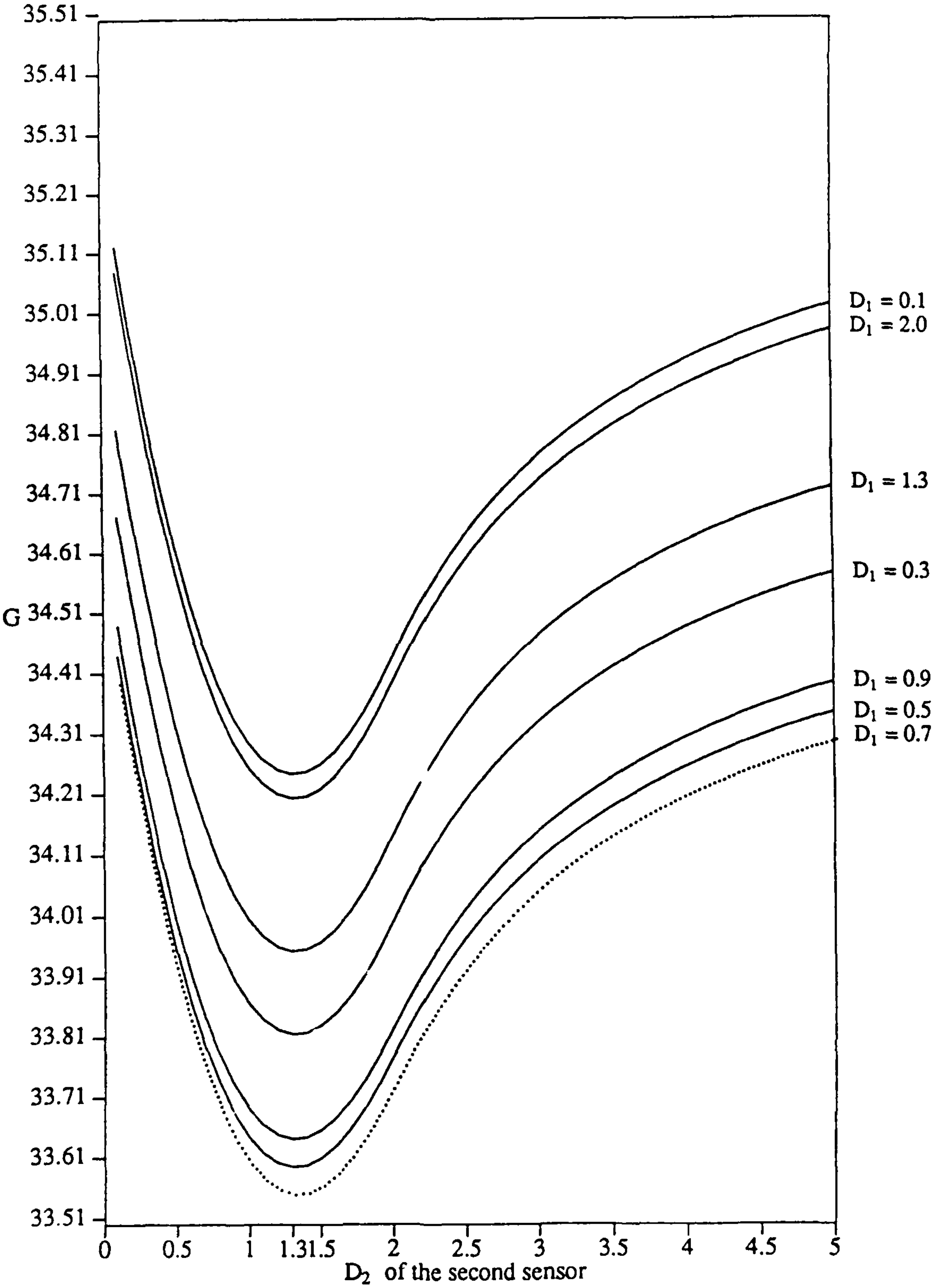

Figure 4.5 G of FQSS with α=.01 β=.05

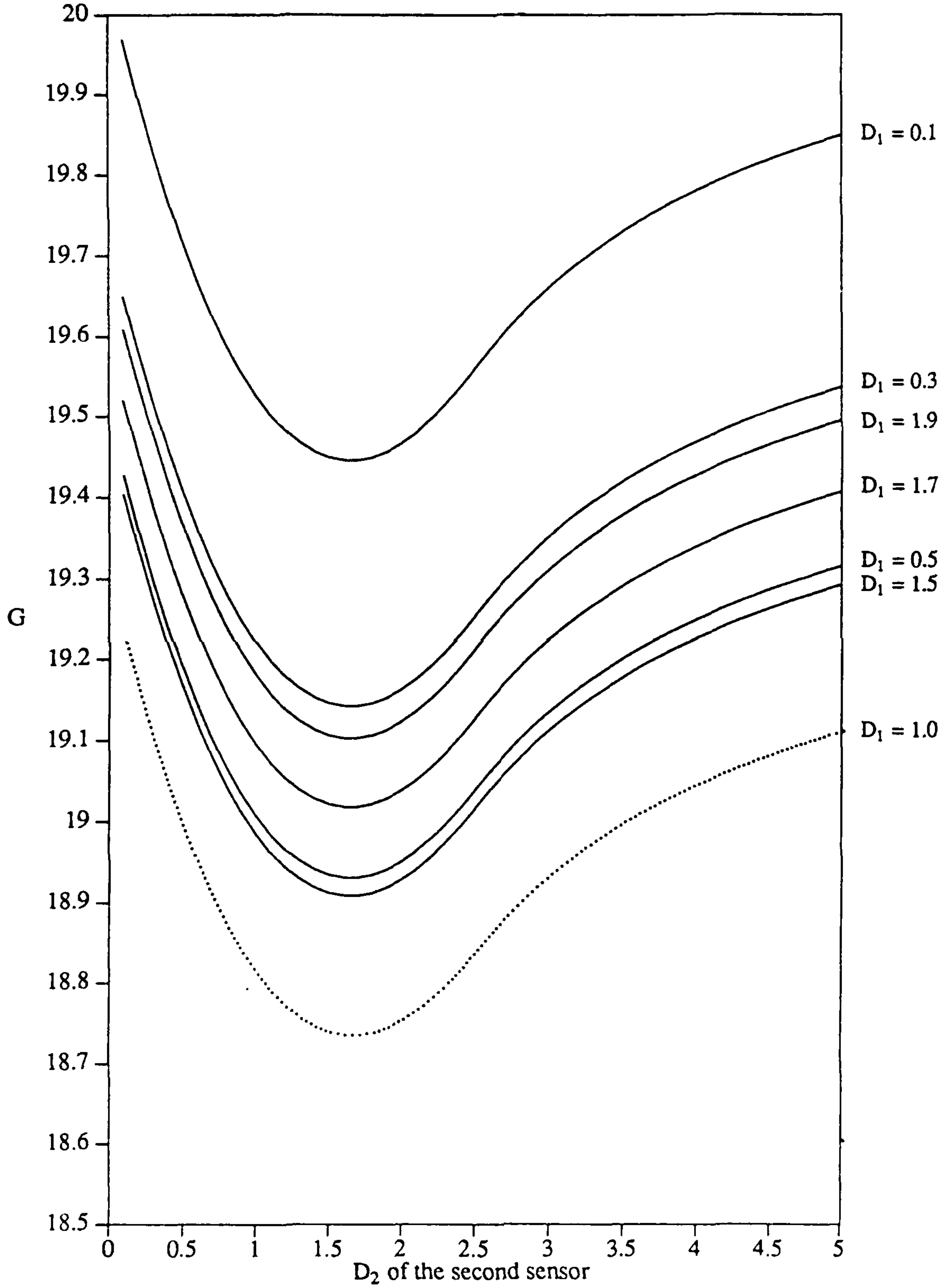

Figure 4.6 G of FQSS with α=.01 β=.05 and means at −2, 1 and −3, 2

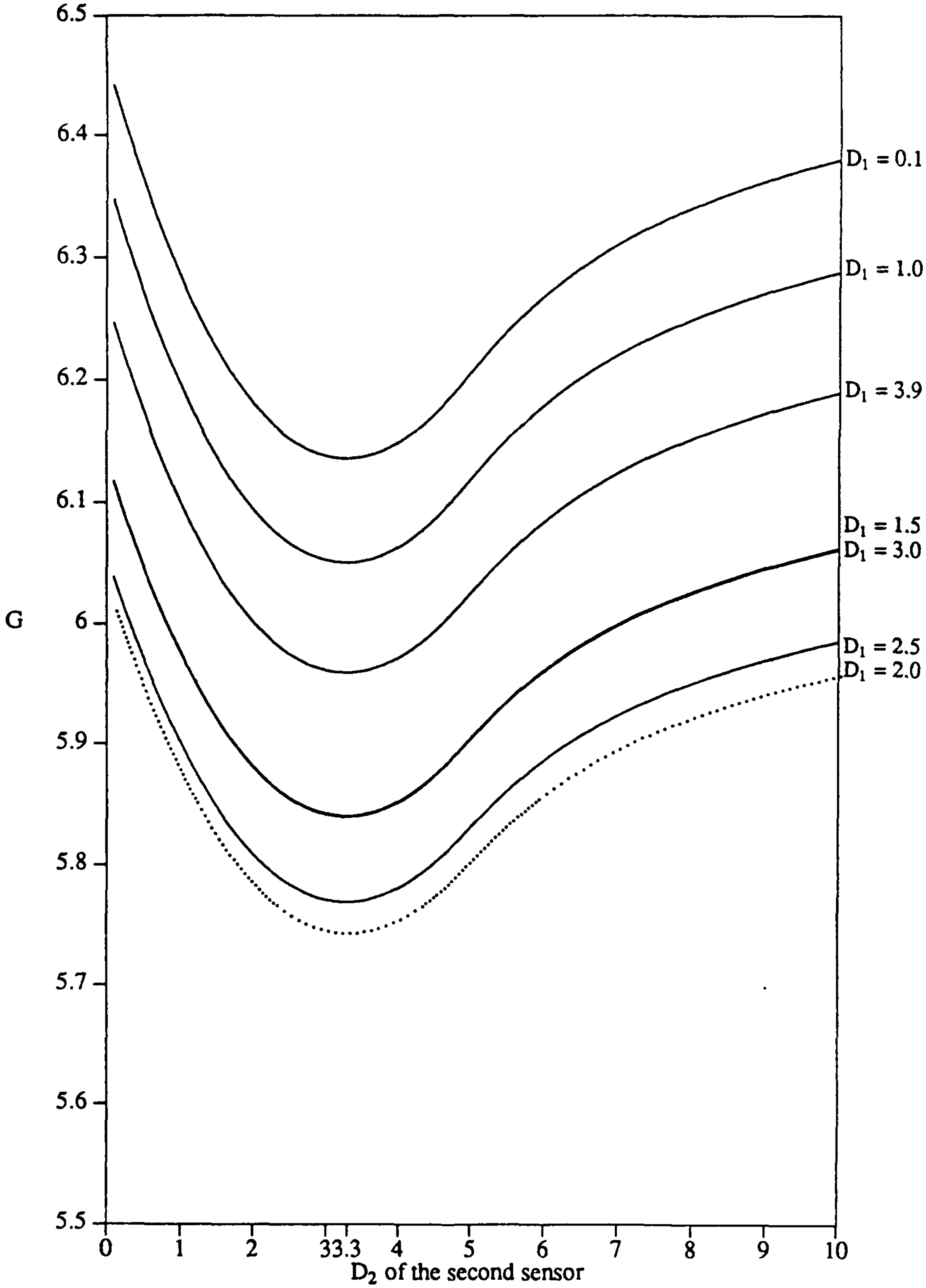

Figure 4.7 G of FQSS with T_1=–.1T_2=–.2 and means at –3,3 and –5,5

CHAPTER 5

Comparison of the Performance of Sequential Systems

5.1. Performance of Individual System

G function has been defined as the sum of the expected number of data to be taken by the system under hypotheses H_1 and H_0. By minimizing G, it also minimizes the data that satisfies both hypotheses. Our optimal sequential system is obtained by finding the minimum of that function.

In the previous chapters, the G function is evaluated over different quantizers, means, and error probabilities. Here, those functions are compared with each other such that different characteristics of the sequential systems are studied.

The plots that we discuss here are from Chapter 3 and 4.

5.1.1. G of Two-level Quantizer System

In Chapter 3, Figure 3.2 shows the G function with respect to T_1 and T_2. As T_1 and T_2 come closer to the origin, G decreases. It has its minima at $T_1 = 0.0$ and $T_2 = 0.0$, and $G = 45.26$. In this case, it seems that T_1 and T_2 are independent of each other that no matter what T_1 is, the minima always occurs at $T_2 = 0.0$. Figure 3.3 shows that G decreases as the error probabilities increase. This can be easily explained since the system allows a larger percentage of error, it requires less data to be collected before making a decision. Again when T_1 and T_2 are at the origin, G has its minima which is equal to 26.63. Figure 3.4 uses different error probabilities,

and $\alpha = 0.01$ and $\beta = 0.05$. The minimum of G is equal to 35.61 at $T_1 = T_2 = 0.0$.

. Table 5.1 lists the the minimum of G from Figure 3.2, Figure 3.3, and Figure 3.4. It is clear that G decreases as the error probabilities increase.

A little more interesting observation can be found when the means move away from the origin as shown in Figure 3.6 and 3.7. The minimum of G does not occur at $T_1 = T_2 = 0.0$. Instead, as in Table 5.2, it has its minima at $T_1 = -0.1$ and $T_2 = -0.2$, and $\alpha > \beta$. The data of Table 5.4 are from Figure 3.6. From the table, one can notice that T_2 at the minimum G increases as T_1 decreases. Figure 3.7 shows the case in which the error probability α is less than β, both T_1 and T_2 are shift to the left, and $T_1 = 0.3$ and $T_2 = 0.2$. The minimum value of G is 6.509 which is smaller than the one in Figure 3.4 under the same error probabilities. This shows that it is easier to detect a signal when the two density functions under hypothesis H_0 and H_1 are further apart.

Thresholds		$\alpha = \beta = 0.01$	$\alpha = 0.01$ $\beta = 0.05$	$\alpha = \beta = 0.05$
T_1	T_2	G	G	G
-0.1	0.0	49.68	39.18	29.23
-0.8	0.0	48.02	37.86	28.26
-0.6	0.0	46.31	36.87	27.53
-0.4	0.0	45.93	36.18	27.03
-0.2	0.0	45.42	35.76	26.73
-0.1	0.0	45.30	35.66	26.66
*0.0	*0.0	45.26	35.61	26.63
0.1	0.0	45.26	35.64	26.66
0.2	0.0	45.42	35.72	26.73
0.4	0.0	45.93	36.10	27.03
0.6	0.0	46.78	36.76	27.53
0.8	0.0	48.02	37.72	28.26
1.0	0.0	49.68	39.00	29.23

Table 5.1 Tabulated Data of Minimum G with Two-level Quantizer

$\alpha = 0.05$ $\beta = 0.01$		
T_1	T_2	G
-0.3	-0.1	8.08
-2.5	-0.1	7.51
-2.0	-0.1	7.16
-1.5	-0.1	6.82
-1.0	-0.1	6.64
-0.8	-0.1	6.59
-0.6	-0.1	6.55
-0.4	-0.2	6.53
-0.2	-0.2	6.517
*-0.1	*-0.2	6.516
0.0	-0.2	6.519
0.1	-0.2	6.52
0.2	-0.2	6.53
0.4	-0.2	6.56
0.8	-0.2	6.64
1.0	-0.2	6.71
1.5	-0.3	6.93
2.0	-0.3	7.24
2.5	-0.3	7.68
2.9	-0.3	8.14

Table 5.2 Minimum G with Two-level Quantizer $\alpha = 0.05$ $\beta = 0.01$

5.1.2. G of the Four-level Quantizer System

In general, as it shows in the graphs of Chapter 4, both D1 and D2 do not change for a given means. However, like previous section, the G function decreases as the error probabilities increase. In Table 5.3, the information in Figure 4.2, Figure 4.4, and Figure 4.5 is tabulated.

As in Figure 4.7, it has the smallest minima of G compared to other figures because the means are further apart from each other which makes the target more distinguishable. This allows the system to require less data to make the decision whether the target is present. The data from Figure 4.7 is tabulated in Table 5.4.

$T_1 = T_2 = 0.0$		$\alpha = \beta = 0.01$	$\alpha = 0.01$ $\beta = 0.05$	
D1	D2	$\alpha = \beta = 0.05$	G	G
0.1	1.3	43.52	34.24	25.61
0.3	1.3	43.00	33.82	25.29
0.5	1.3	42.69	33.59	25.12
0.6	1.3	42.64	33.55	25.09
*0.7	*1.3	42.63	33.548	25.088
0.8	1.3	42.67	33.58	25.11
1.0	1.3	42.86	33.73	25.22
1.5	1.3	43.27	34.05	25.46
2.0	1.3	43.49	34.20	25.58
3.0	1.3	43.65	34.45	25.69
4.0	1.3	43.74	34.42	25.74

Table 5.3 Tabulated Data of Minimum G with Four-level Quantizer

$T_1 = -0.1$ $T_2 = -0.2$		$\alpha = 0.05$ $\beta = 0.01$
D1	D2	G
0.1	3.3	6.14
0.5	3.3	6.78
1.0	3.3	5.84
1.5	3.3	5.76
1.7	3.3	5.75
1.9	3.3	5.743
*2.0	*3.3	5.742
2.1	3.3	5.744
2.3	3.3	5.75
2.5	3.3	5.77
3.0	3.3	5.84
4.0	3.3	5.97
5.0	3.3	6.04

Table 5.4 Minimum G with Four-level Quantizer, $\alpha = 0.05$ $\beta = 0.01$

Table 5.5 tabulates the data of various T_1 and T_2 with $\alpha = 0.01$, $\beta = 0.01$, $u_{11} = -1$, $u_{12} = 1$, $u_{21} = -2$, and $u_{22} = 2$. This shows how G varies with T_1 and T_2.

5.2. Comparison of System

From the numerical analysis, one can conclude that the system with a four-level quantizer, in average, requires less data to make a decision than the one with a two-level quantizer. By compared by picking a column from both Table 5.1 and Table

	$T_1 = -0.2$ $T_2 = -0.3$		$T_1 = T_2 = 0.0$		$T_1 = 0.3$ $T_2 = 0.5$	
D2	D1	G	D1	G	D1	G
0.1	0.7	43.84	0.7	43.70	0.8	44.06
0.3	0.7	43.49	0.7	43.36	0.8	43.69
0.5	0.7	43.21	0.7	43.09	0.8	43.39
0.6	0.7	43.10	0.7	42.98	0.8	43.27
0.8	0.7	42.92	0.7	42.81	0.8	43.08
1.0	0.7	42.80	0.7	42.70	0.8	42.94
1.5	0.7	42.73	0.7	42.65	0.8	42.84
2.0	0.7	42.90	0.7	42.87	0.8	43.94
3.0	0.7	43.33	0.7	43.27	0.8	43.39
4.0	0.7	43.55	0.7	43.48	0.8	43.67

Table 5.5 G with Various T_1 and T_2

5.3. G of the two-level system is larger than the one of the four-level system. For $\alpha = 0.01$, $\beta = 0.01$, $u_{11} = -1$, $u_{12} = 1$, $u_{21} = -2$, and $u_{22} = 2$, the improvement on the G function is 5.8 %. However, for $\alpha = 0.05$, $\beta = 0.01$, $u_{11} = -3$, $u_{12} = 3$, $u_{21} = -5$, and $u_{22} = 5$, the improvement on the G function is 11.88 %. Table 5.6 shows the improvement on the G function for various error probabilities and means.

Notice from Table 5.6, the improvement of G for a given set of means are the same no matter what the error probabilities.

The other observation is that T_1 and T_2 of the four-level quantizer system is the same of the two-level quantizer system. T_1 and T_2 from a two-level quantizer system can be used for designing a four-level quantizer if it is under same means and error probabilities. This approach will reduce the total number of calculation to find the optimal G.

Condition	G with Two-level Quantizer	G with Four-level Quantizer	% Improvement
$\alpha = 0.01$ $\beta = 0.01$ $u_{11} = -1$ $u_{12} = 1$ $u_{21} = -2$ $u_{22} = 2$	45.26	42.63	5.8 %
$\alpha = 0.01$ $\beta = 0.05$ $u_{11} = -1$ $u_{12} = 1$ $u_{21} = -2$ $u_{22} = 2$	26.63	25.09	5.8 %
$\alpha = 0.05$ $\beta = 0.05$ $u_{11} = -1$ $u_{12} = 1$ $u_{21} = -2$ $u_{22} = 2$	35.61	33.55	5.8 %
$\alpha = 0.05$ $\beta = 0.01$ $u_{11} = -3$ $u_{12} = 3$ $u_{21} = -5$ $u_{22} = 5$	6.516	5.742	11.88 %

Table 5.6 Improvement on G

CHAPTER 6

System Simulations

6.1. Simulation Method

From Chapter 3 and 4, the optimal sequential detection systems are found. It has been shown that the quantized levels of a quantizer has a major effect on the sequential system. By selecting the quantizers that associate with the optimal system, the system can be evaluated under the real environment.

6.1.1. Derivation of The Double Exponential Environment

Since the computer that is used to simulate the system has a random value generator with uniform distribution, transformation method must be used to generate the double exponential density environment.

Let x be the uniform distributed random variable, and $x \in (0, 1)$, then the double exponential density function $F_y(y)$ is equal to x. Thus $y = F_y^{-1}(x)$.

The distribution function can be evaluated by integration of the density function $f_y(x) = \frac{\lambda}{2} e^{-\lambda|x-\mu|}$, and

$$F_y(x) = \int_{-\infty}^{x} \frac{\lambda}{2} e^{-\lambda|z-\mu|} dz$$

If $x \geq \mu$, then

$$F_y(x) = 1 - \frac{1}{2} e^{-\lambda(x-\mu)}$$

If $x < \mu$, then

$$F_y(y) = \frac{1}{2}e^{-\lambda(\mu - x)}$$

the inverse fuction $F_y(y)$ is

$$y = \mu + \frac{1}{\lambda}\log(2x) \qquad \text{for} \quad x \geq 0.5$$

$$y = \mu - \frac{1}{\lambda}\log(2(1-x)) \qquad \text{for} \quad x < 0.5$$

6.1.2. Simulation and Discussion

The first systems to be simulated are under the conditions that $\mu_{11} = -1$, $\mu_{12} = 1$, $\mu_{21} = -2$, and $\mu_{22} = 2$. With the error probabilities varing, the amount of data N required to make a decision are collected. Total of one thousand trials are made, and Figure 6.1 shows N of two-level quantizer system with error probabilities $\alpha = 0.01$ and $\beta = 0.01$. Figure 6.2 is N of four-level quantizer system with same error probabilities as in Figure 6.2. In Figure 6.3, Figure 6.4, and Figure 6.5, it gives an enlarged view on how N varies in different trial and error probabilities.

Figure 6.6 assumes that the means $\mu_{11} = -3$, $\mu_{12} = 3$, $\mu_{21} = -5$, and $\mu_{22} = 5$, and N under both quantized systems are plotted. Notice that the dashed line of all plots represents N of the four-level quantizer system.

From the graphs, one may notice that the system with four-level quantizer has a higher G values than the system with two-level one. The results of the system simulations confirm that in average the four-level quantizer system requirs less amount of data than the two-level one.

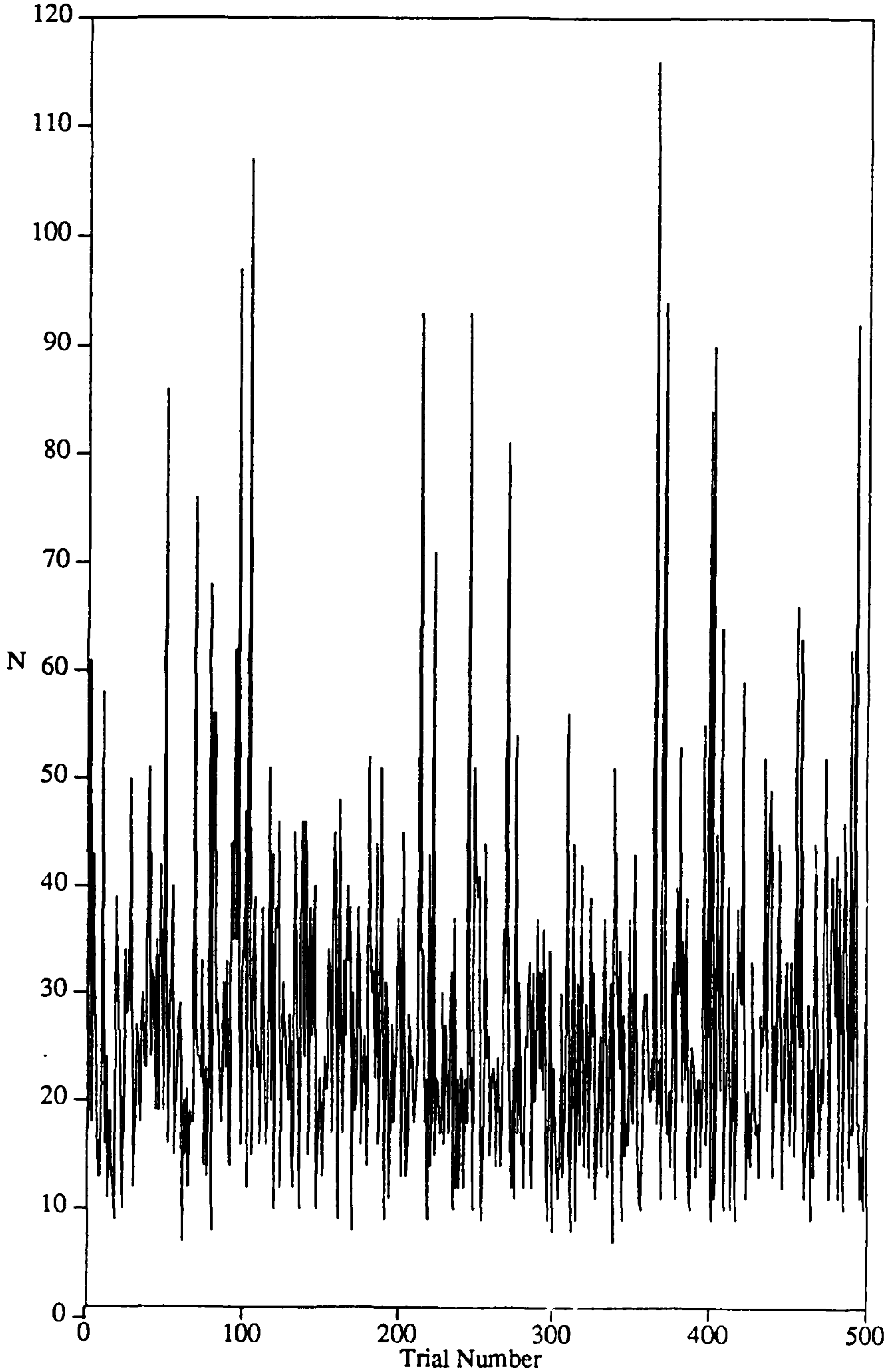

Figure 6.1 N of Two-level Quantizer System

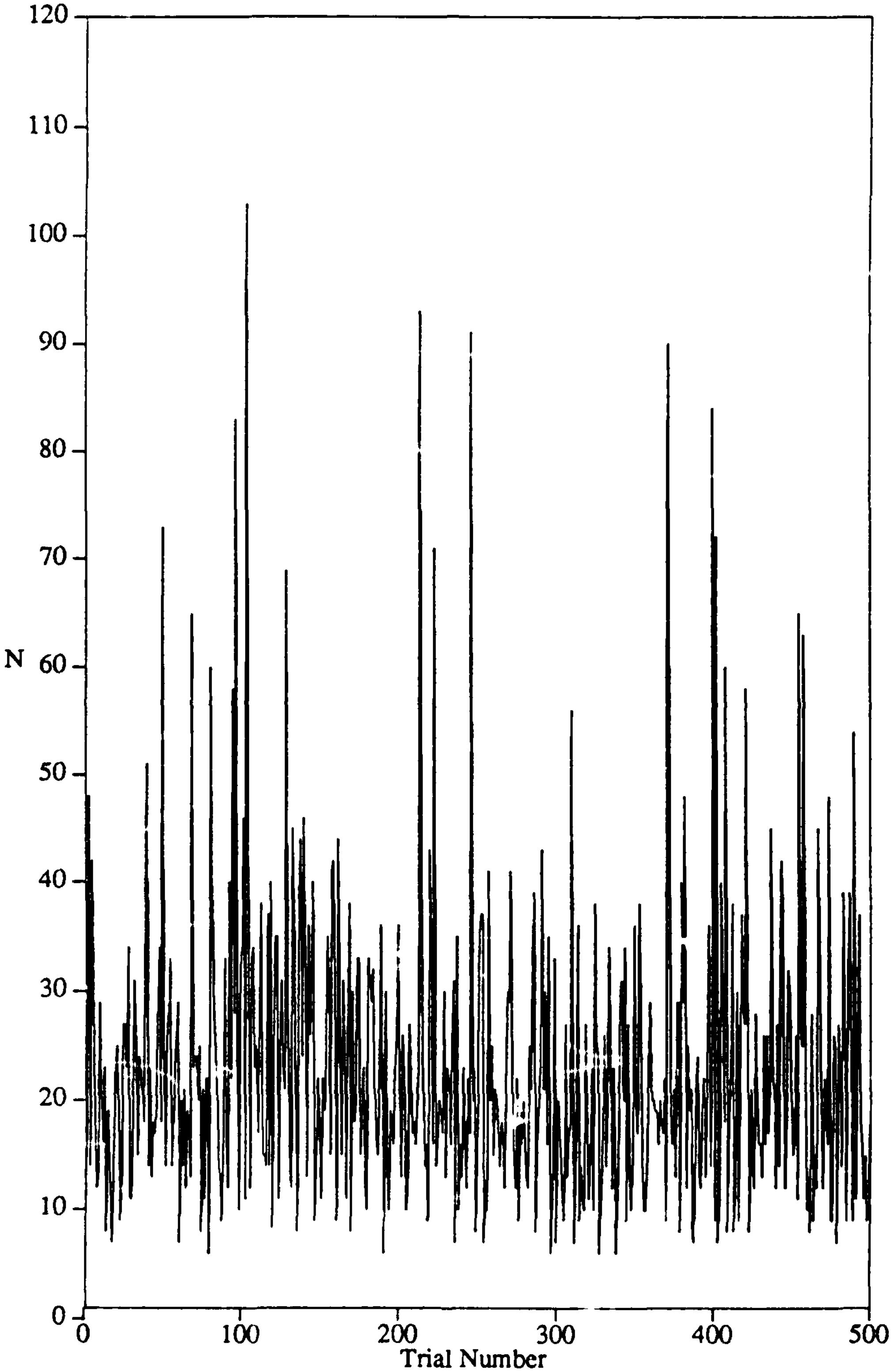

Figure 6.2 N of Four-level Quantizer System

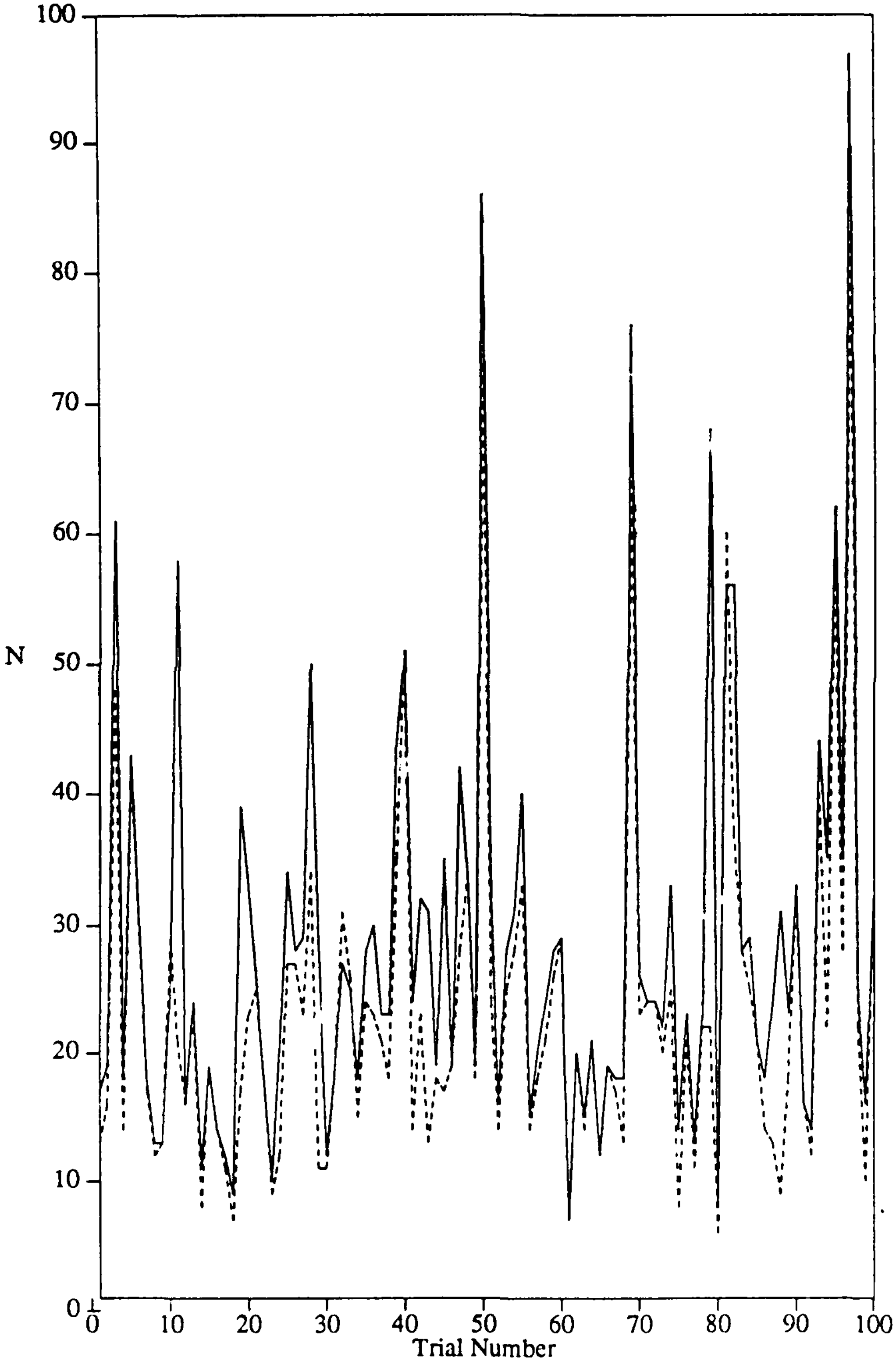

Figure 6.3 Enlarged N with $\alpha = \beta = 0.01$

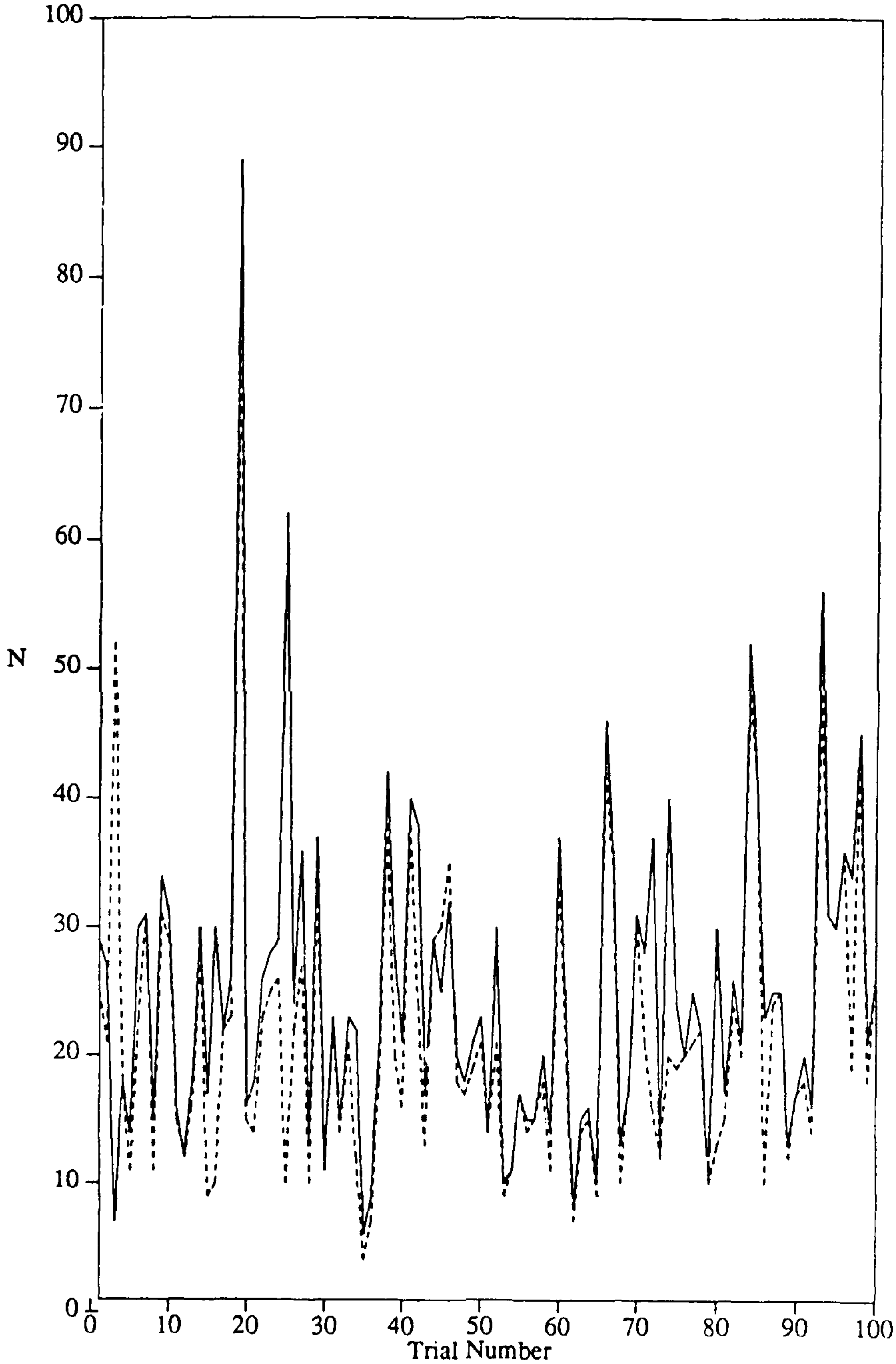

Figure 6.4 Enlarged N with $\alpha = 0.01$ $\beta = 0.05$

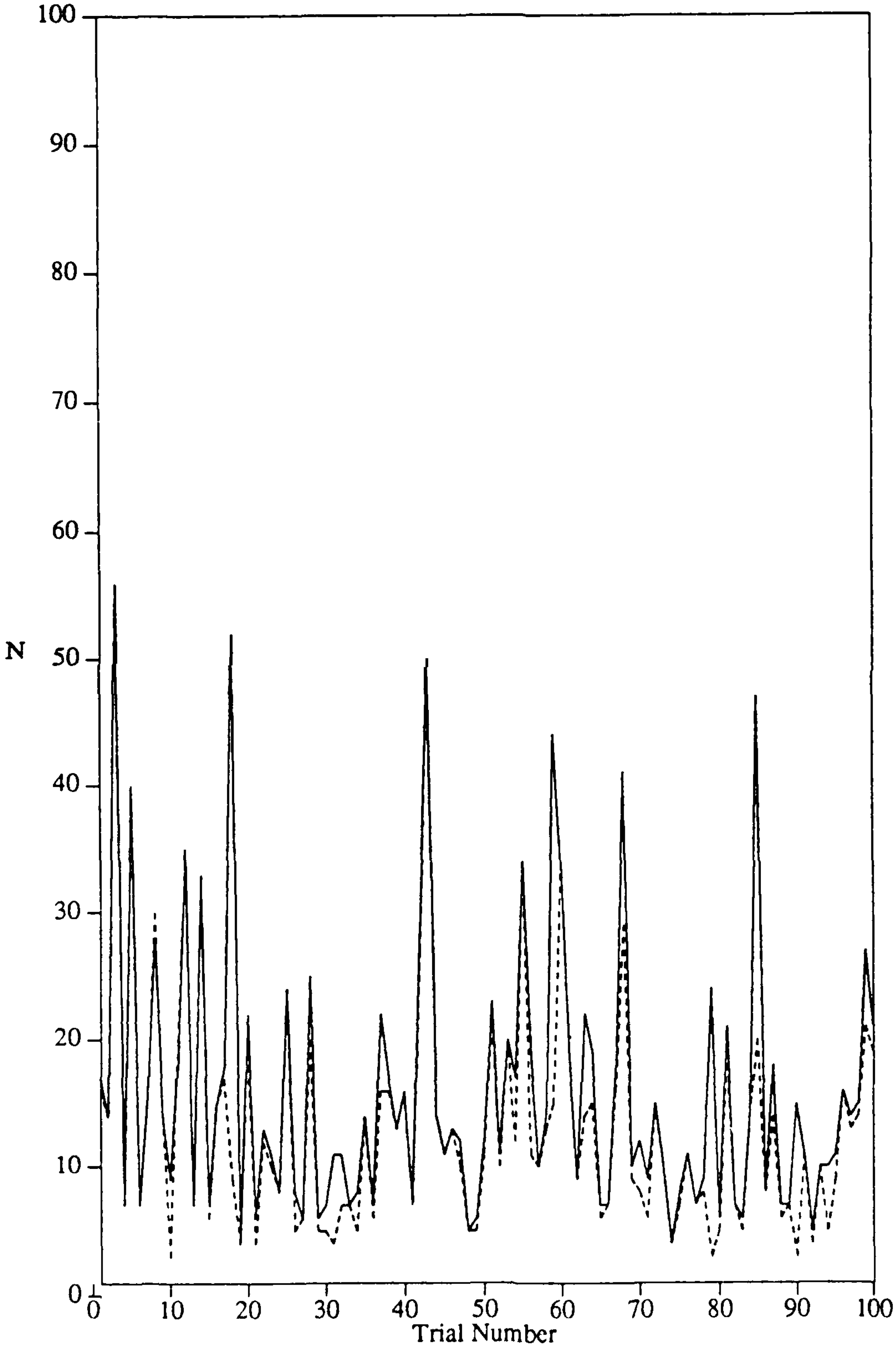

Figure 6.5 Enlarged N with $\alpha = 0.05$ $\beta = 0.05$

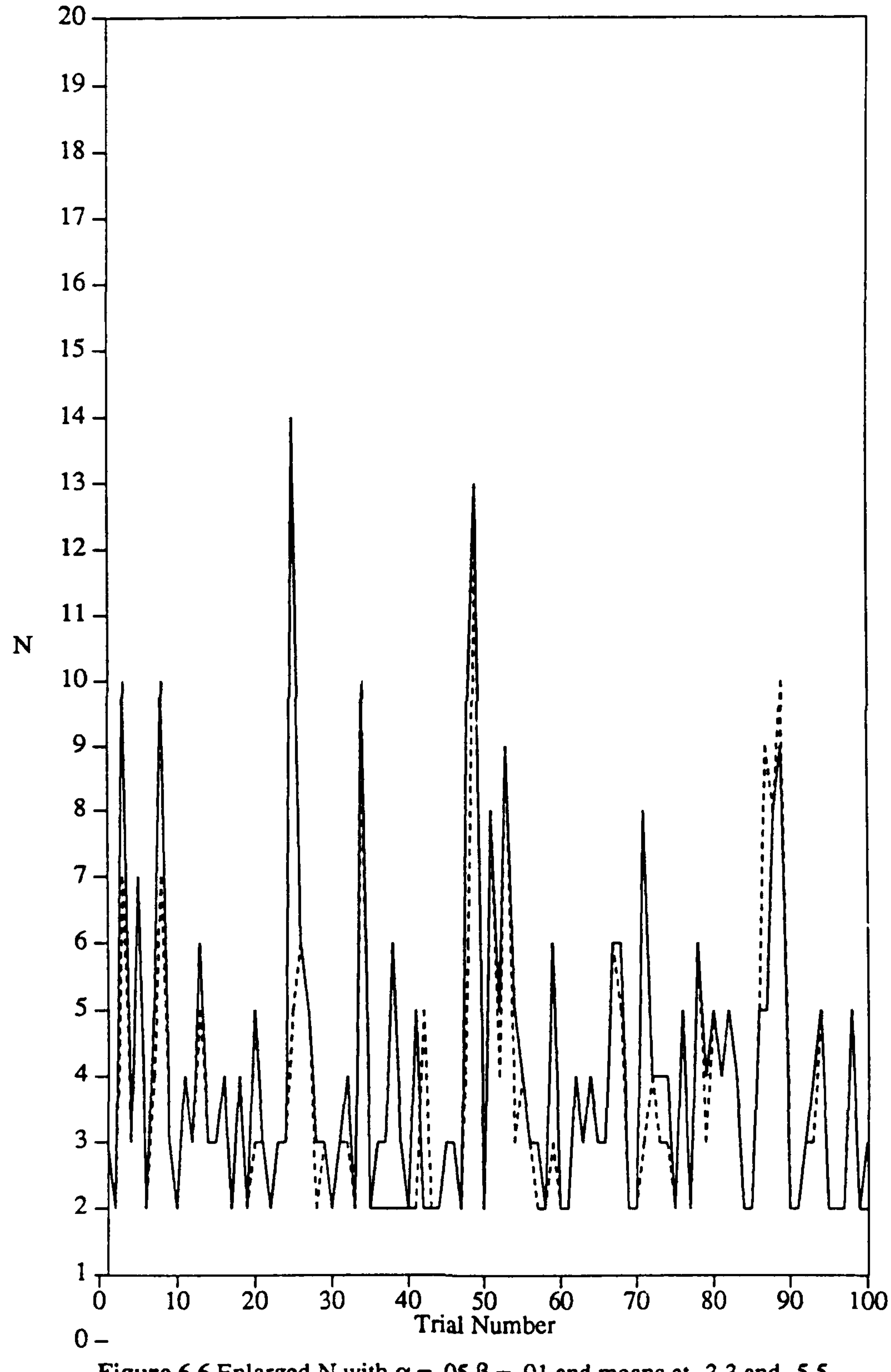

Figure 6.6 Enlarged N with $\alpha = .05$ $\beta = .01$ and means at -3,3 and -5,5

CHAPTER 7

Conclusion

The objective of this study is to analysis and design a two-sensor sequential detection system. The optimal system is found by numerical analysis, and that system is simulated.

In this study, the expected number of observation is first derived, and then function G is defined as the sum of that expected number under both hypotheses. By minimizing G, it minimizes the expected value of number of observations required.

G is directly related to the quantizer of each sensor. As shown in chapter 2, function G is an equation in terms of the probability of the quantized levels. Minimum G can be found by selecting appropriate quantizer.

Two-level and four-level quantized system are used in the analysis. Assuming the environment is double exponentially distributed, G is evaluated under various conditions. The numerical evaluations show that a four-level quantizer system has smaller value of G than a two-level one. This shows that a four-level quantizer system requires less data than a two-level one to make a decision.

It is also found that for a given density function, the quantizers are always the same for both sensors to have a minimum value of G no matter what are the error probabilities. As the means of the density function moving further apart, G decreases, and the quantized levels are not necessarily divide the probability space symmetrically. The system simulation confirmed the results of the evaluation.

In the future, it might be interesting to look into the case of dependent observation.

APPENDIX A

Program for Evaluation of G Function of Two-level Quantizer System

This program calculates the G function of two-level quantizer system for various parameters. The means of double exponential and error probabilities are provided, and then this program calculate the probabilities associated with each quantized level for different two-level quantizers. The main routine find the value of G.

The plots in chapter 3 are provided by this program.

```
/*****************************************************************
  This program is used to find the minimun number of data that
  required to determine wheter a target is present by
  appling sequencial detection theory. Two independent sensers
  are used to detect the signals from the source. Two-level
  quantizers are used here, and optimal quantizer is found.
*****************************************************************/ #include
<stdio.h> #include <math.h> /*define the value of error probabilities*/ #define alpha 0.05 #define beta
0.01 /*define the constants of double exponential density functions*/ #define U10 -3 #define U11 3
#define U20 -5 #define U21 5 #define l1 0.25 #define l2 0.125

main() /* find min EN and store the data for further uses*/ {
 double T1,T2;  /*quantized levels of both sensors*/
 double Cons1(),Cons0();
 double U1(),U2();
 double P1(),P2();
 double f;
 double m,ta,tb;
 double U();
 FILE *fpout;
 fpout = fopen("chp3fg6","w");
/************************************************************ * calculate G func-
tion * ************************************************************/
 for ( T1 = -3.0; T1 <= 3.0; T1 = T1 + 0.1) {
  for ( T2 = -5.0; T2 <= 5.0; T2 = T2 + 0.1) {
   f = (Cons1() / U1(T1,T2)) + (Cons0() / U2(T1,T2));
   fprintf(fpout,"%5.2f %7.2f %10.4f0,T1,T2,f);
  }

 }
 fclose(fpout); }

double U(p1,p0,q1,q0,c1,c2) /* calculate the U expression */ double p1,p0,q1,q0; double c1,c2; {
 double i;
 i =  c1 * log((p1 * (1 - p0)) / (p0 * (1 - p1)))
    + c2 * log((q1 * (1 -q0)) / (q0 * (1 - q1)))
    + log(((1 - p1) * (1 - q1)) / ((1 - p0) * (1 - q0)));
 return(i); }

/* find the probabilities of quantized levels*/ double P1(l,t,u) double l,t,u; {
 double i;
 i = 0.5 * exp((-l) * (t - u));
 return(i); }

double P2(l,t,u) double l,t,u; {
```

```
double i;
i = 1 - 0.5 * exp( (-l) * (u - t));
return(i); }

/* find the value of U1 which involves prob p & q */ double U1(t1,t2) double t1,t2; {
double p1,p0,q1,q0;
double c1,c2;
double P1(),P2();
double U();
double u11,u10,u21,u20;
double L1,L2;
u11 = U11;
u10 = U10;
u21 = U21;
u20 = U20;
L1  = l1;
L2  = l2;

  if (t1 >= u11 && t2 >= u21) {
   p1 = P1(L1,t1,u11);
   p0 = P1(L1,t1,u10);
   q1 = P1(L2,t2,u21);
   q0 = P1(L2,t2,u20);
   c1 = p1;
   c2 = q1;
   return( U(p1,p0,q1,q0,c1,c2));
  }
  if (t1 >= u11 && t2 >= u20 && t2 < u21) {
   p1 = P1(L1,t1,u11);
   p0 = P1(L1,t1,u10);
   q1 = P2(L2,t2,u21);
   q0 = P1(L2,t2,u20);
   c1 = p1;
   c2 = q1;
   return( U(p1,p0,q1,q0,c1,c2));
  }
  if (t1 >= u11 && t2 < u20) {
   p1 = P1(L1,t1,u11);
   p0 = P1(L1,t1,u10);
   q1 = P2(L2,t2,u21);
   q0 = P2(L2,t2,u20);
   c1 = p1;
   c2 = q1;
   return( U(p1,p0,q1,q0,c1,c2));
  }
```

```
if (t1 < u11 && t1 >= u10 && t2 >= u21) {
 p1 = P2(L1,t1,u11);
 p0 = P1(L1,t1,u10);
 q1 = P1(L2,t2,u21);
 q0 = P1(L2,t2,u20);
 c1 = p1;
 c2 = q1;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u11 && t1 >= u10 && t2 < u21 && t2 >= u20) {
 p1 = P2(L1,t1,u11);
 p0 = P1(L1,t1,u10);
 q1 = P2(L2,t2,u21);
 q0 = P1(L2,t2,u20);
 c1 = p1;
 c2 = q1;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u11 && t1 >= u10 && t2 < u20) {
 p1 = P2(L1,t1,u11);
 p0 = P1(L1,t1,u10);
 q1 = P2(L2,t2,u21);
 q0 = P2(L2,t2,u20);
 c1 = p1;
 c2 = q1;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u10 && t2 >= u21) {
 p1 = P2(L1,t1,u11);
 p0 = P2(L1,t1,u10);
 q1 = P1(L2,t2,u21);
 q0 = P1(L2,t2,u20);
 c1 = p1;
 c2 = q1;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u10 && t2 < u21 && t2 >= u20) {
 p1 = P2(L1,t1,u11);
 p0 = P2(L1,t1,u10);
 q1 = P2(L2,t2,u21);
 q0 = P1(L2,t2,u20);
 c1 = p1;
 c2 = q1;
 return( U(p1,p0,q1,q0,c1,c2));
}
```

```
  if (t1 < u10 && t2 < u20) {
    p1 = P2(L1,t1,u11);
    p0 = P2(L1,t1,u10);
    q1 = P2(L2,t2,u21);
    q0 = P2(L2,t2,u20);
    c1 = p1;
    c2 = q1;
    return( U(p1,p0,q1,q0,c1,c2));
  } }

/* find the value of U2 in function G which involves prob p & q */ double U2(t1,t2) double t1,t2; {
 double p1,p0,q1,q0;
 double c1,c2;
 double P1(),P2();
 double U();
 double u11,u10,u21,u20;
 double L1,L2;
 u11 = U11;
 u10 = U10;
 u21 = U21;
 u20 = U20;
 L1 = l1;
 L2 = l2;


  if (t1 >= u11 && t2 >= u21) {
    p1 = P1(L1,t1,u11);
    p0 = P1(L1,t1,u10);
    q1 = P1(L2,t2,u21);
    q0 = P1(L2,t2,u20);
    c1 = p0;
    c2 = q0;
    return( U(p1,p0,q1,q0,c1,c2));
  }
  if (t1 >= u11 && t2 >= u20 && t2 < u21) {
    p1 = P1(L1,t1,u11);
    p0 = P1(L1,t1,u10);
    q1 = P2(L2,t2,u21);
    q0 = P1(L2,t2,u20);
    c1 = p0;
    c2 = q0;
    return( U(p1,p0,q1,q0,c1,c2));
  }
  if (t1 >= u11 && t2 < u20) {
    p1 = P1(L1,t1,u11);
```

```
 p0 = P1(L1,t1,u10);
 q1 = P2(L2,t2,u21);
 q0 = P2(L2,t2,u20);
 c1 = p0;
 c2 = q0;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u11 && t1 >= u10 && t2 >= u21) {
 p1 = P2(L1,t1,u11);
 p0 = P1(L1,t1,u10);
 q1 = P1(L2,t2,u21);
 q0 = P1(L2,t2,u20);
 c1 = p0;
 c2 = q0;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u11 && t1 >= u10 && t2 < u21 && t2 >= u20) {
 p1 = P2(L1,t1,u11);
 p0 = P1(L1,t1,u10);
 q1 = P2(L2,t2,u21);
 q0 = P1(L2,t2,u20);
 c1 = p0;
 c2 = q0;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u11 && t1 >= u10 && t2 < u20) {
 p1 = P2(L1,t1,u11);
 p0 = P1(L1,t1,u10);
 q1 = P2(L2,t2,u21);
 q0 = P2(L2,t2,u20);
 c1 = p0;
 c2 = q0;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u10 && t2  >= u21) {
 p1 = P2(L1,t1,u11);
 p0 = P2(L1,t1,u10);
 q1 = P1(L2,t2,u21);
 q0 = P1(L2,t2,u20);
 c1 = p0;
 c2 = q0;
 return( U(p1,p0,q1,q0,c1,c2));
}
if (t1 < u10 && t2 < u21 && t2 >= u20) {
 p1 = P2(L1,t1,u11);
```

```
  p0 = P2(L1,t1,u10);
  q1 = P2(L2,t2,u21);
  q0 = P1(L2,t2,u20);
  c1 = p0;
  c2 = q0;
  return( U(p1,p0,q1,q0,c1,c2));
 }
 if (t1 < u10 && t2 < u20) {
  p1 = P2(L1,t1,u11);
  p0 = P2(L1,t1,u10);
  q1 = P2(L2,t2,u21);
  q0 = P2(L2,t2,u20);
  c1 = p0;
  c2 = q0;
  return( U(p1,p0,q1,q0.c1,c2));
 } }

/*calculate constants in function G*/ double Cons1() {
 double a,b;
 double A,B;
 double i;
 A = (beta) / (1 - alpha);
 B = (1 - beta)/ alpha;
 a = log(A);
 b = log(B);
 i = (a * A * (B - 1) + b * B * (1 - A)) / (B - A);
 return(i); }

double Cons0() {
 double a,b;
 double A,B;
 double i;
 A = (beta) / (1 - alpha);
 B = (1 - beta) / alpha;
 a = log(A);
 b = log(B);
 i = (a * (B - 1) + b * (1 - A)) / (B - A);
 return(i); }
```

APPENDIX B

Program for Evaluation of G Function of Four-level Quantizer System

This program calculates the G function of four-level quantizer system for various parameters. The means of double exponential and error probabilities are provided, and then this program calculate the probabilities associated with each quantized level for different four-level quantizers. The main routine find the value of G.

The plots in chapter 4 are provided by this program.

```
/*******************************************************************
  This program is used to find the minimum number of data that
  required to determine whether a target is present by
  appling sequencial detection theory.  Two independent sensers
  are used to detect the signals from the source. G function is
  calculated            using            various            four-level            quantizers.
*********************************************************************/           #include
<stdio.h> #include <math.h> #define alpha 0.05 #define beta  0.01 #define U10 -3 #define U11 3
#define U20 -5 #define U21 5 #define I1  0.25 #define I2  0.125 #define T1  -0.1 #define T2  -0.0

main() /* find min G and store the data for further uses*/ {
 double D1,D2;
 double Cons1(),Cons0();
 double U1();
 double P1(),P2(),P3(),P4(),P5(),P6(),P7();
 double f;
 double m,d3,d4;
 double U();
 int   k;
 FILE *fpout,*fp;

 m = 1000.0;
 d3 = 0.0;
 d4 = 0.0; /********************************************************* *        calculate G
function           * *********************************************************/
 fpout = fopen("chap4fig6.t","w");
 fp = fopen("supfig6.t","w");
 for ( D1 = 0.1; D1 <= 3.0; D1 = D1 + 0.1) {
  for ( D2 = 0.1; D2 <= 5.0; D2  = D2 + 0.1) {
   f = (Cons1() / U1(D1,D2,1)) + (Cons0() / U1(D1,D2,0));
   if ( f <= m) {
    m = f;
    d3 = D1;
    d4 = D2; .
   }
   fprintf(fpout,"%5.2f %7.2f %10.4f0,D1,D2,f);
  }
 }
 fprintf(fp,"%5.2f %5.2f %10.4f0,d3,d4,m);
 fclose(fpout);
 fclose(fp); }

/*********************************************************************/

double  U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,k) /* calculate the U
```

```
expression */ double p41,p31,p21,p11,p40,p30,p20,p10; double q41,q31,q21,q11,q40,q30,q20,q10; int
k; {
 double i,a,b;
 if(k == 0){
  a = p10 * q10 * log((p11 * q11) / (p10 * q10))
    + p20 * q10 * log((p21 * q11) / (p20 * q10))
    + p30 * q10 * log((p31 * q11) / (p30 * q10))
    + p40 * q10 * log((p41 * q11) / (p40 * q10))
    + p10 * q20 * log((p11 * q21) / (p10 * q20))
    + p20 * q20 * log((p21 * q21) / (p20 * q20))
    + p30 * q20 * log((p31 * q21) / (p30 * q20))
    + p40 * q20 * log((p41 * q21) / (p40 * q20));
  b = p10 * q30 * log((p11 * q31) / (p10 * q30))
    + p20 * q30 * log((p21 * q31) / (p20 * q30))
    + p30 * q30 * log((p31 * q31) / (p30 * q30))
    + p40 * q30 * log((p41 * q31) / (p40 * q30))
    + p10 * q40 * log((p11 * q41) / (p10 * q40))
    + p20 * q40 * log((p21 * q41) / (p20 * q40))
    + p30 * q40 * log((p31 * q41) / (p30 * q40))
    + p40 * q40 * log((p41 * q41) / (p40 * q40));
  i = a + b;
   return(i);
  }
  if(k==1){
   a = p11 * q11 * log((p11 * q11) / (p10 * q10))
     + p21 * q11 * log((p21 * q11) / (p20 * q10))
     + p31 * q11 * log((p31 * q11) / (p30 * q10))
     + p41 * q11 * log((p41 * q11) / (p40 * q10))
     + p11 * q21 * log((p11 * q21) / (p10 * q20))
     + p21 * q21 * log((p21 * q21) / (p20 * q20))
     + p31 * q21 * log((p31 * q21) / (p30 * q20))
     + p41 * q21 * log((p41 * q21) / (p40 * q20));
   b = p11 * q31 * log((p11 * q31) / (p10 * q30))
     + p21 * q31 * log((p21 * q31) / (p20 * q30))
     + p31 * q31 * log((p31 * q31) / (p30 * q30))
     + p41 * q31 * log((p41 * q31) / (p40 * q30))
     + p11 * q41 * log((p11 * q41) / (p10 * q40))
     + p21 * q41 * log((p21 * q41) / (p20 * q40))
     + p31 * q41 * log((p31 * q41) / (p30 * q40))
     + p41 * q41 * log((p41 * q41) / (p40 * q40));
   i = a + b;
   return(i);
  } }

/*******************************************************/
```

```
/*    find the probabilities of quantized levels   */ double P1(u,l,t) double l,t,u; {
 double i;
 i = 0.5 * exp((-l) * (t - u));
 return(i); }

double P2(u,l,t,d) double u,l,t,d; {
 double i;
  i = 1 - 0.5 * exp((-l) * (u - t)) - 0.5 * exp((-l) * (t + d - u));
 return(i); }

double P3(u,l,t) double l,t,u; {
 double i;
 i = 0.5 * exp((-l) * (u - t));
 return(i); }

double P4(u,l,t) double l,t,u; {
 double i;
 i = 1 - 0.5 * exp( (-l) * (u - t));
 return(i); }

double P5(u,l,t,d) double u,l,t,d; {
 double i;
  i = - 0.5 * exp((-l) * (u - t)) + 0.5 * exp((-l) * (u - t - d));
 return(i); }

double P6(u,l,t,d) double u,l,t,d; {
 double i;
  i =  0.5 * exp((-l) * (t - u)) - 0.5 * exp((-l) * (t + d - u));
 return(i); }

double P7(u,l,t) double l,t,u; {
 double i;
 i = 1 - 0.5 * exp( (-l) * (t - u));
 return(i); }

/*******************************************************/

/* find the value of U1 which involves prob p & q */ double U1(d1,d2,k) double d1,d2; int   k; {
 double p11,p21,p31,p41;
 double p10,p20,p30,p40;
 double q11,q21,q31,q41;
 double q10,q20,q30,q40;
 double P1(),P2(),P3(),P4(),P5(),P6(),P7();
 double U();
 double u11,u10,u21,u20;
```

```
double L1,L2;
double t1,t2;
t1 = T1;
t2 = T2;
u11 = U11;
u10 = U10;
u21 = U21;
u20 = U20;
L1  = l1;
L2  = l2;

 if ((t1+d1) < u11 && (t1-d1) >= u10 && (t2+d2) < u21 && (t2-d2) >= u20)
 {
  p41 = P4(u11,L1,t1+d1);
  p31 = P5(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P6(u10,L1,t1-d1,d1);
  p10 = P7(u10,L1,t1-d1);

  q41 = P4(u21,L2,t2+d2);
  q31 = P5(u21,L2,t2,d2);
  q21 = P5(u21,L2,t2-d2,d2);
  q11 = P3(u21,L2,t2-d2);
  q40 = P1(u20,L2,t2+d2);
  q30 = P6(u20,L2,t2,d2);
  q20 = P6(u20,L2,t2-d2,d2);
  q10 = P7(u20,L2,t2-d2);

  return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
      k));
 }
 if((t1+d1)<u11 && (t1-d1)>= u10 && (t2+d2)<u21 && (t2-d2)<u20 )
 {
  p41 = P4(u11,L1,t1+d1);
  p31 = P5(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P6(u10,L1,t1-d1,d1);
  p10 = P7(u10,L1,t1-d1);
```

```
 q41 = P4(u21,L2,t2+d2);
 q31 = P5(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P2(u20,L2,t2-d2,d2);
 q10 = P3(u20,L2,t2-d2);
 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
}
if((t1+d1)<u11 && (t1-d1)>= u10 && (t2+d2)>=u21 && (t2-d2)>=u20)
{
 p41 = P4(u11,L1,t1+d1);
 p31 = P5(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P6(u10,L1,t1-d1,d1);
 p10 = P7(u10,L1,t1-d1);

 q41 = P1(u21,L2,t2+d2);
 q31 = P2(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P6(u20,L2,t2-d2,d2);
 q10 = P7(u20,L2,t2-d2);
 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
}
if((t1+d1)<u11 && (t1-d1)>= u10 && (t2+d2)>=u21 && (t2-d2) < u20)
{
 p41 = P4(u11,L1,t1+d1);
 p31 = P5(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P6(u10,L1,t1-d1,d1);
 p10 = P7(u10,L1,t1-d1);

 q41 = P1(u21,L2,t2+d2);
```

```
 q31 = P2(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P2(u20,L2,t2-d2,d2);
 q10 = P3(u20,L2,t2-d2);
 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
} /*******************************************************/

if ((t1+d1) < u11 && (t1-d1) < u10 && (t2+d2) < u21 && (t2-d2) >= u20)
{
 p41 = P4(u11,L1,t1+d1);
 p31 = P5(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P2(u10,L1,t1-d1,d1);
 p10 = P3(u10,L1,t1-d1);

 q41 = P4(u21,L2,t2+d2);
 q31 = P5(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P6(u20,L2,t2-d2,d2);
 q10 = P7(u20,L2,t2-d2);

 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
}
if((t1+d1)<u11 && (t1-d1)< u10 && (t2+d2)<u21 && (t2-d2)<u20 )
{
 p41 = P4(u11,L1,t1+d1);
 p31 = P5(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P2(u10,L1,t1-d1,d1);
 p10 = P3(u10,L1,t1-d1);
```

```
  q41 = P4(u21,L2,t2+d2);
  q31 = P5(u21,L2,t2,d2);
  q21 = P5(u21,L2,t2-d2,d2);
  q11 = P3(u21,L2,t2-d2);
  q40 = P1(u20,L2,t2+d2);
  q30 = P6(u20,L2,t2,d2);
  q20 = P2(u20,L2,t2-d2,d2);
  q10 = P3(u20,L2,t2-d2);
  return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
      k));
 }
 if((t1+d1)<u11 && (t1-d1) < u10 && (t2+d2)>=u21 && (t2-d2)>=u20)
 {
  p41 = P4(u11,L1,t1+d1);
  p31 = P5(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P2(u10,L1,t1-d1,d1);
  p10 = P3(u10,L1,t1-d1);

  q41 = P1(u21,L2,t2+d2);
  q31 = P2(u21,L2,t2,d2);
  q21 = P5(u21,L2,t2-d2,d2);
  q11 = P3(u21,L2,t2-d2);
  q40 = P1(u20,L2,t2+d2);
  q30 = P6(u20,L2,t2,d2);
  q20 = P6(u20,L2,t2-d2,d2);
  q10 = P7(u20,L2,t2-d2);
  return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
      k));
 }
 if((t1+d1)<u11 && (t1-d1) < u10 && (t2+d2)>=u21 && (t2-d2)<u20 )
 {
  p41 = P4(u11,L1,t1+d1);
  p31 = P5(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P2(u10,L1,t1-d1,d1);
  p10 = P3(u10,L1,t1-d1);

  q41 = P1(u21,L2,t2+d2);
```

```
 q31 = P2(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P2(u20,L2,t2-d2,d2);
 q10 = P3(u20,L2,t2-d2);
 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
} /*****************************************************************/

if ((t1+d1) >= u11 && (t1-d1) >= u10 && (t2+d2) < u21 && (t2-d2) >= u20)
{
 p41 = P1(u11,L1,t1+d1);
 p31 = P2(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P6(u10,L1,t1-d1,d1);
 p10 = P7(u10,L1,t1-d1);

 q41 = P4(u21,L2,t2+d2);
 q31 = P5(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P6(u20,L2,t2-d2,d2);
 q10 = P7(u20,L2,t2-d2);

 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
}
if((t1+d1)>=u11 && (t1-d1)>= u10 && (t2+d2)<u21 && (t2-d2)<u20 )
{
 p41 = P1(u11,L1,t1+d1);
 p31 = P2(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P6(u10,L1,t1-d1,d1);
 p10 = P7(u10,L1,t1-d1);
```

```
  q41 = P4(u21,L2,t2+d2);
  q31 = P5(u21,L2,t2,d2);
  q21 = P5(u21,L2,t2-d2,d2);
  q11 = P3(u21,L2,t2-d2);
  q40 = P1(u20,L2,t2+d2);
  q30 = P6(u20,L2,t2,d2);
  q20 = P2(u20,L2,t2-d2,d2);
  q10 = P3(u20,L2,t2-d2);
  return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
      k));
 }
 if((t1+d1)>=u11 && (t1-d1) >= u10 && (t2+d2)>=u21 && (t2-d2)>=u20)
 {
  p41 = P1(u11,L1,t1+d1);
  p31 = P2(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P6(u10,L1,t1-d1,d1);
  p10 = P7(u10,L1,t1-d1);

  q41 = P1(u21,L2,t2+d2);
  q31 = P2(u21,L2,t2,d2);
  q21 = P5(u21,L2,t2-d2,d2);
  q11 = P3(u21,L2,t2-d2);
  q40 = P1(u20,L2,t2+d2);
  q30 = P6(u20,L2,t2,d2);
  q20 = P6(u20,L2,t2-d2,d2);
  q10 = P7(u20,L2,t2-d2);
  return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
      k));
 }
 if((t1+d1) >= u11 && (t1-d1) >= u10 && (t2+d2)>=u21 && (t2-d2)<u20 )
 {
  p41 = P1(u11,L1,t1+d1);
  p31 = P2(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P6(u10,L1,t1-d1,d1);
  p10 = P7(u10,L1,t1-d1);

  q41 = P1(u21,L2,t2+d2);
```

```
 q31 = P2(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P2(u20,L2,t2-d2,d2);
 q10 = P3(u20,L2,t2-d2);
 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
} /*******************************************************************/

if ((t1+d1) >= u11 && (t1-d1) < u10 && (t2+d2) < u21 && (t2-d2) >= u20)
{
 p41 = P1(u11,L1,t1+d1);
 p31 = P2(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P2(u10,L1,t1-d1,d1);
 p10 = P3(u10,L1,t1-d1);

 q41 = P4(u21,L2,t2+d2);
 q31 = P5(u21,L2,t2,d2);
 q21 = P5(u21,L2,t2-d2,d2);
 q11 = P3(u21,L2,t2-d2);
 q40 = P1(u20,L2,t2+d2);
 q30 = P6(u20,L2,t2,d2);
 q20 = P6(u20,L2,t2-d2,d2);
 q10 = P7(u20,L2,t2-d2);

 return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
     k));
}
if((t1+d1)>=u11 && (t1-d1) < u10 && (t2+d2)<u21 && (t2-d2)<u20 )
{
 p41 = P1(u11,L1,t1+d1);
 p31 = P2(u11,L1,t1,d1);
 p21 = P5(u11,L1,t1-d1,d1);
 p11 = P3(u11,L1,t1-d1);
 p40 = P1(u10,L1,t1+d1);
 p30 = P6(u10,L1,t1,d1);
 p20 = P2(u10,L1,t1-d1,d1);
 p10 = P3(u10,L1,t1-d1);
```

```
  q41 = P4(u21,L2,t2+d2);
  q31 = P5(u21,L2,t2,d2);
  q21 = P5(u21,L2,t2-d2,d2);
  q11 = P3(u21,L2,t2-d2);
  q40 = P1(u20,L2,t2+d2);
  q30 = P6(u20,L2,t2,d2);
  q20 = P2(u20,L2,t2-d2,d2);
  q10 = P3(u20,L2,t2-d2);
  return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
      k));
}
if((t1+d1)>=u11 && (t1-d1) < u10 && (t2+d2)>=u21 && (t2-d2)>=u20)
{
  p41 = P1(u11,L1,t1+d1);
  p31 = P2(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P2(u10,L1,t1-d1,d1);
  p10 = P3(u10,L1,t1-d1);

  q41 = P1(u21,L2,t2+d2);
  q31 = P2(u21,L2,t2,d2);
  q21 = P5(u21,L2,t2-d2,d2);
  q11 = P3(u21,L2,t2-d2);
  q40 = P1(u20,L2,t2+d2);
  q30 = P6(u20,L2,t2,d2);
  q20 = P6(u20,L2,t2-d2,d2);
  q10 = P7(u20,L2,t2-d2);
  return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
      k));
}
if((t1+d1) >= u11 && (t1-d1) < u10 && (t2+d2)>=u21 && (t2-d2)<u20 )
{
  p41 = P1(u11,L1,t1+d1);
  p31 = P2(u11,L1,t1,d1);
  p21 = P5(u11,L1,t1-d1,d1);
  p11 = P3(u11,L1,t1-d1);
  p40 = P1(u10,L1,t1+d1);
  p30 = P6(u10,L1,t1,d1);
  p20 = P2(u10,L1,t1-d1,d1);
  p10 = P3(u10,L1,t1-d1);

  q41 = P1(u21,L2,t2+d2);
```

```
    q31 = P2(u21,L2,t2,d2);
    q21 = P5(u21,L2,t2-d2,d2);
    q11 = P3(u21,L2,t2-d2);
    q40 = P1(u20,L2,t2+d2);
    q30 = P6(u20,L2,t2,d2);
    q20 = P2(u20,L2,t2-d2,d2);
    q10 = P3(u20,L2,t2-d2);
    return( U(p41,p31,p21,p11,p40,p30,p20,p10,q41,q31,q21,q11,q40,q30,q20,q10,
        k));
  } }

/*****************************************************/

/*calculate constants in the G function */ double Cons1() {
 double a,b;
 double A,B;
 double i;
 A = (beta) / (1 - alpha);
 B = (1 - beta) / alpha;
 a = log(A);
 b = log(B);
 i = (a * A * (B - 1) + b * B * (1 - A)) / (B - A);
 return(i); }

double Cons0() {
 double a,b;
 double A,B;
 double i;
 A = (beta) / (1 - alpha);
 B = (1 - beta) / alpha;
 a = log(A);
 b = log(B);
 i = (a * (B - 1) + b * (1 - A)) / (B - A);
 return(i); }
```

APPENDIX C

Program for Evaluation of G Function of Two-level Quantizer System

This program is to generate double exponential random environment, and it is then be used to simulate the sequential detection system with two and four level quantization. The number of data that have to be collected to determine which signal has been received is calculated. This program makes comperison between 2 and 4 level quatizers.

Since these two sensors are independent, the random generator is started twice in order to make the environment of sensor one independent from the one of sensor two.

```
#define RND(seed)  rand(seed) / 32767.  #define RND(weed)  rand(weed) / 32767.
#define max 500 #include <math.h> #include <stdio.h> int rand(); void srand();
unsigned int seed; unsigned int weed;

main() {
 float y1,y2;
 float lamda,lamda1;
 float u0,u1,U0,U1;
 float a11,a12,a01,a02; /* quantized areas for 2 level quantizer */
 float ar11,ar12,ar13,ar14,ar01,ar02,ar03,ar04; /* quantized areas for 4 level*/
 float A11,A12,A01,A02;
 float Ar11,Ar12,Ar13,Ar14,Ar01,Ar02,Ar03,Ar04;
 float t1,t2;
 float T1,T2,Delta1, Delta2;
 float f1, f0;
 float F1, F0;
 float ff1, ff0;
 float FF1, FF0;
 float A,B;
 float miss1,miss2;
 float test1,test2;
 float arriv1(), arriv2();
 float Area1(), Area2(),Area3(), Area4(), Area5(), Area6(), Area7();
 float alpha, beta;
 int  i,j,k,m,n;
 int  ok1, ok2;
 int  N1[max+1];
 int  N2[max+1];
 FILE *fp1, *fp2, *fp3, *fp4;

/* initial the random number for sensor 1     */
 seed = time(&seed);
 srand(seed); /*Given the constants of double exponential function*/
 lamda = 0.25;
 lamda1 = 0.125;
 u0 = -3.0;
 u1 = 3.0;
 U0 = -5.0;
 U1 = 5.0; /*define the optimal quantizers of for sensor 1 and 2 */
 t1 = 0.0;
 t2 = 0.0;
 T1 = 0.0;
 T2 = 0.0;
 Delta1 = 2.0;
 Delta2 = 3.3;
```

```
alpha = 0.05;
beta = 0.01; /*calculate the threshold of sequential test*/
A = beta / ( 1 - alpha );
B = ( 1 - beta ) / alpha ;
fp1 = fopen("num1fig4","w");
fp2 = fopen("num2fig4","w");
fp3 = fopen("mis1fig4","w");
fp4 = fopen("mis2fig4","w"); /*find the probabilities of each quantized level for a
given */ /* quantizer                                    */
a11 = Area2(lamda,u1,t1);
a12 = Area3(lamda,u1,t1);
a01 = Area4(lamda,u0,t1);
a02 = Area1(lamda,u0,t1);
A11 = Area2(lamda1,U1,t2);
A12 = Area3(lamda1,U1,t2);
A01 = Area4(lamda1,U0,t2);
A02 = Area1(lamda1,U0,t2);
if (( T1 + Delta1) >= u1 )
{
 ar11 = Area2(lamda,u1,T1-Delta1);
 ar12 = Area5(lamda,u1,T1-Delta1,T1);
 ar13 = Area7(lamda,u1,T1,Delta1+T1);
 ar14 = Area1(lamda,u1,T1+Delta1);
 ar01 = Area2(lamda,u0,T1-Delta1);
 ar02 = Area7(lamda,u0,T1-Delta1,T1);
 ar03 = Area6(lamda,u0,T1,T1+Delta1);
 ar04 = Area1(lamda,u0,T1+Delta1);
}
if (( T1 + Delta1 ) < u1 )
{
 ar11 = Area2(lamda,u1,T1-Delta1);
 ar12 = Area5(lamda,u1,T1-Delta1,T1);
 ar13 = Area5(lamda,u1,T1,Delta1+T1);
 ar14 = Area3(lamda,u1,T1+Delta1);
 ar01 = Area4(lamda,u0,T1-Delta1);
 ar02 = Area6(lamda,u0,T1-Delta1,T1);
 ar03 = Area6(lamda,u0,T1,T1+Delta1);
 ar04 = Area1(lamda,u0,T1+Delta1);
}
if( (T2 + Delta2) >= U1 )
{
 Ar11 = Area2(lamda1,U1,T2-Delta2);
 Ar12 = Area5(lamda1,U1,T2-Delta2,T2);
 Ar13 = Area7(lamda1,U1,T2,T2+Delta2);
 Ar14 = Area1(lamda1,U1,T2+Delta2);
```

```
 Ar01 = Area2(lamda1,U0,T2-Delta2);
 Ar02 = Area7(lamda1,U0,T2-Delta2,T2);
 Ar03 = Area6(lamda1,U0,T2,T2+Delta2);
 Ar04 = Area1(lamda1,U0,T2+Delta2);
 }
if( (T2 + Delta2) < U1 )
{
 Ar11 = Area2(lamda1,U1,T2-Delta2);
 Ar12 = Area5(lamda1,U1,T2-Delta2,T2);
 Ar13 = Area5(lamda1,U1,T2,T2+Delta2);
 Ar14 = Area3(lamda1,U1,T2+Delta2);
 Ar01 = Area4(lamda1,U0,T2-Delta2);
 Ar02 = Area6(lamda1,U0,T2-Delta2,T2);
 Ar03 = Area6(lamda1,U0,T2,T2+Delta2);
 Ar04 = Area1(lamda1,U0,T2+Delta2);
 } /* initial the random number for sensor 2      */
weed = time(&weed);
srand(weed);

m = 0;
n = 0;
for(i=1; i<= max; i++)
{
 j = 0;
 k = 0;
 ok1 = 0;
 ok2 = 0;
 test1 = 1.0;
 test2 = 1.0;
 while((ok1 == 0) || (ok2 == 0))
 {
  y1 = arriv1(lamda,u1);
  y2 = arriv2(lamda1,U1);

/* calculate the number of collected data from two-level quantizer */
  if(ok1 == 0)
  {

   if(y1 >= t1)
   {
    f1 = a12;
    f0 = a02;
   }
   else if (y1 <= t1)
   {
```

```
  f1 = a11;
  f0 = a01;
 }
 if(y2 >= t2)
 {
  F1 = A12;
  F0 = A02;
 }
 else if (y2<= t2)
 {
  F1 = A11;
  F0 = A01;
 }
 test1 = ((f1 * F1) / (f0 * F0))*test1;
 if((test1 >= B) || (test1 <= A))
  ok1 = 1;
 if(test1 <= A)
 {
  m++;
 }
 j++;
} /* calculate the number of collected data from four-level quantizer */
if(ok2 == 0)
{
 if(y1 < T1 - Delta1)
 {
  ff1 = ar11;
  ff0 = ar01;
 }
 if(y1 >= (T1 - Delta1) && y1 < T1)
 {
  ff1 = ar12;
  ff0 = ar02;
 }
 if(y1 >= T1 && y1 < (T1 + Delta1))
 {
  ff1 = ar13;
  ff0 = ar03;
 }
 if(y1 >= (T1 + Delta1))
 {
  ff1 = ar14;
  ff0 = ar04;
 }
 if(y2 < (T2 - Delta2))
```

```
{
 FF1 = Ar11;
 FF0 = Ar01;
}
if(y2 >= (T2 - Delta2) && y2 < T2)
{
 FF1 = Ar12;
 FF0 = Ar02;
}
if(y2 >= T2 && y2 < (T2 + Delta2))
{
 FF1 = Ar13;
 FF0 = Ar03;
}
if(y2 >= (T2 + Delta2))
{
 FF1 = Ar14;
 FF0 = Ar04;
}
test2 = ((ff1 * FF1) / (ff0 * FF0))*test2;
if((test2 >= B) || (test2 <= A))
 ok2 = 1;
if(test2 <= A)
 n++;
k++;
}
}
N1[i] = j;
N2[i] = k;
fprintf(fp1,"%d %d0,i,N1[i]);
fprintf(fp2,"%d %d0,i,N2[i]);
}
fclose(fp1);
fclose(fp2);
fclose(fp3);
fclose(fp4); } /* generate the enviornment H1 for sensor 1   */ float arriv1(l, u )
float l, u;

{
 float z,r;
 int  k,y;
 double x;

  x = RND(seed);
  if (x < 0.5 && x > 0.0)
```

```
  {
   y = 1;
   z = u + (y * log(2 * x)) / l ;
   return(z);
  }
  else if ( x >= 0.5 && x < 1.0)
  {
   y = -1;
   z = u + (y * log ( 2 * ( 1 - x ))) / l ;
   return(z);
  } } /* generate the enviornment H1 for sensor 2    */ float arriv2(l, u ) float l, u;

{
 float z,r;
 int  k,y;
 double x;

  x = RND(weed);
  if (x < 0.5 && x > 0.0)
  {
   y = 1;
   z = u + (y * log(2 * x)) / l ;
   return(z);
  }
  else if (x >= 0.5 && x < 1.0 )
  {
   y = -1;
   z = u + y * (log ( 2 * ( 1 - x ))) / l ;
   return(z);
  } }

/* calculate the probabilies of different quantized region for */ /* two and four-level
quantizer                    */

float Area1(l,u,x) float l,u,x; {
 return(0.5 * exp(-l * (x - u))); } float Area2(l,u,x) float l,u,x; {
 return(0.5 * exp(-l * (u - x))); } float Area3(l,u,x) float l,u,x; {
 return(1 - 0.5 * exp(-l * (u - x))); } float Area4(l,u,x) float l,u,x; {
 return(1 - 0.5 * exp(-l * (x - u))); } float Area5(l,u,x,y) float l,u,x,y; {
 return(0.5 * exp(-l * (u - y)) - 0.5 * exp(-l * (u - x))); } float Area6(l,u,x,y) float
l,u,x,y; {
 return(0.5 * exp(-l * (x - u)) - 0.5 * exp(-l * (y - u))); } float Area7(l,u,x,y) float
l,u,x,y; {
 return(1 - 0.5 * exp(-l * (u - x)) - 0.5 * exp(-l * (y - u))); }
```

References

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